SET-A (Heat Transfer) ①

(Code: 5ME1A)

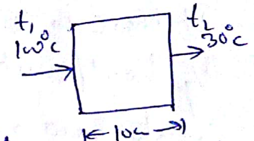
Ans 1 (a) Heat is a form of Energy. It is always in a dynamic state; The flow of heat is due to the temperature difference. Heat always flows from a body at higher temperature to a body at lower temperature itself. There are three modes of heat transfer. ① Conduction ② Convection ③ Radiation. Transfer of heat energy from one place to other place is due to the existance of temperature gradiant $\frac{dT}{dx}$.

Thermodynamics deals with equilibrium of system. It tells us the direction of heat transfer. Transfer of heat from one equilibrium state to other equilibrium state can be estimated by the use of basic thermodynamic laws. But the rate of heat transfer and temperature variation with space and time are not ~~est~~ to estimated through thermodynamic laws. For the calculation we need the help of Heat Transfer fundamental laws. Heat transfer requires a temperature gradient i.e we must be aware of temperature distribution within a body. This is all possible through the study of subject Heat Transfer.

(b) Temp gradient $= \frac{dt}{dx} = \frac{30-100}{10}$

$= \frac{-70°C}{10} = -7°C/cm$

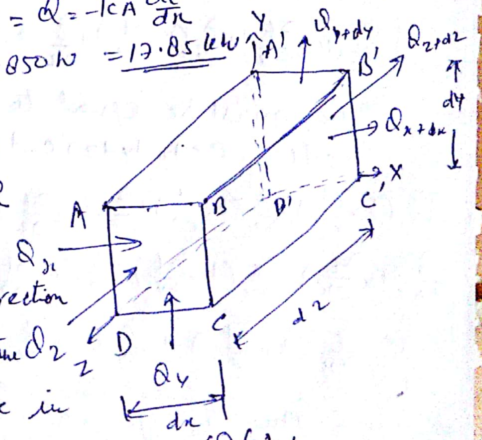
$= -7 \times 10^2\ °C/m = -700°C/m$



Heat conduction across the wall $= Q = -kA\frac{dt}{dx}$

$Q = -0.5 \times 3 \times (-700) = 17850\ W = 17.85\ kW$

OR



Consider a small elemental vol of volume $dx.dy.dz$

As temperature changes along x-direction and rate of change of temperature Q_z is given by $\frac{\partial t}{\partial x}$, then change in temperature through dx distance is given by $\left(\frac{\partial t}{\partial x}\right)dx$

∴ Temperature at face $AA'D'D$ is if t at face $BB'C'C$ it will be $\left[t + \left(\frac{\partial t}{\partial x}\right)dx\right]$

If thermal conductivities of material are k_x, k_y & k_z along x, y & z directions respectively. The amount of heat flowing into control volume ~~ttoo~~ through this face during $d\tau$ time is

$$Q_x = -k_x(dy.dz)\frac{\partial t}{\partial x}.d\tau \quad —①$$

Heat outflow from face $BB'C'C$ during this time is

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x)dx \qquad \therefore dQ_x = Q_x - \left[Q_x + \frac{\partial}{\partial x}(Q_x)dx\right]$$

$$\therefore dQ_x = -\frac{\partial}{\partial x}(Q_x)dx = -\left[\frac{\partial}{\partial x}\left\{-k_x(dydz)\frac{\partial t}{\partial x}.\partial\tau\right]dx$$

or $$dQ_x = \frac{\partial}{\partial x}\left[k_x\frac{\partial t}{\partial x}\right](dxdydz)\,d\tau. \quad —(ii)$$

Similarly

$$dQ_y = \frac{\partial}{\partial y}\left[k_y\frac{\partial t}{\partial y}\right](dxdydz)\,d\tau \quad —(iii)$$

& $$dQ_z = \frac{\partial}{\partial z}\left[k_z\frac{\partial t}{\partial z}\right](dxdydz)\,d\tau \quad —(iv)$$

$\therefore$ Total net heat accumulated in elemental volume $= Q_x + Q_y + Q_z$

$$= \left[\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)\right](dxdydz)\,d\tau \quad — (A)$$

If there is a heat source inside control volume which generates q_g heat/unit vol/unit time then heat generation $= q_g(dxdydz)d\tau$

$\therefore$ Total heat accumulation inside control volume is

$$\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)\right](dxdydz)\,d\tau + q_g(dx.dy.dz)\,d\tau$$

This must be equal to $mC\,dt$, if ρ is the density of material

$mcdt = \rho(dxdydz)\,c.dt$, so from heat balance

$$\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)\right](\cancel{dxdydz})\,\frac{}{d\tau} + q_g(\cancel{dxdydz})d\tau = \rho(\cancel{dxdydz})\,c\,dt$$

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right) + q_g = \rho c\frac{dt}{\partial\tau}$$

$$\therefore \boxed{\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right) + q_g = \rho.c.\frac{dt}{\partial\tau}}$$

This is general conduction eqn. in Cartesian coordinate.

Keeping $k_x = k_y = k_z = k$ for homogeneous materials

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho c}{k}\frac{dt}{\partial\tau} = \frac{1}{\alpha}\frac{dt}{\partial\tau}$$

, where α is thermal diffusivity $\boxed{\alpha = \frac{k}{\rho c}}$

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Heat loss with insulation $Q_1 = \frac{t_1 - t_0}{R_{t_1} + R_{t_2}}$

$R_{t_1} = \frac{1}{2\pi k l} \ln \frac{r_c}{r_i} = \frac{1}{2\pi \times 0.18 \times 1.5} \ln \frac{21.2}{4} = 0.98 \, K/W$ (Kelvin/W)

$R_{t_2} = \frac{1}{2\pi r_c h_0} = \frac{1}{2\pi \times (21.2 \times 10^{-3}) \times 8.5} = 0.000884 \times 10^3 = 0.884 \, K/W$

$\therefore Q_1 = \frac{60-22}{0.98+0.884} = \frac{38}{1.864} = 20.386 \, W$

Heat loss w/o insulation $Q = hA(t_s - t_a)$

$Q = 8.5 \times (2\pi r_i l) \times (60-22) = 8.5 \times 2 \times 3.14 \times 4 \times 10^{-3} \times 1.5 \times 38$

$Q = 12.17 \, W$

$\therefore$ % increase $= \frac{20.386 - 12.17}{12.17} \times 100 = 67.51\%$.

Q.3(a) At many places from the surfaces it is needed to dissipate the heat at a faster rate to its surroundings in order to keep the temperature of surface within limit. The heat flow rate becomes the measure of cooling the surface. According to Newton's law of cooling $Q = hA(t_s - t_a)$ the value of Q can be increased by increasing the area A. So the surfaces are extended by the ~~proto~~ protrusions to these surfaces and so the heat transfer rate is substantially improved. The protrusions are called fins or splines.

Applications. (1) Air cooled I.C. engines and air cooled cylinder of aircraft engines & air compressors. (2) Electric transformers & motors (3) cooling coils and condenser coils in Refrigeration and air conditioners etc.

(b) Total area for heat flow.

$A = 2[(10 \times 2) + (8 \times 2)] + 10 \times 8 = 144 \, m^2$

$R_t = \frac{1}{A}\left[\frac{\delta_1}{k_1} + \frac{\delta_2}{k_2} + \frac{\delta_3}{k_3}\right] = \frac{1}{144}\left[\frac{60 \times 10^{-3}}{0.2} + \frac{90 \times 10^{-3}}{0.04} + \frac{240 \times 10^{-3}}{1.8}\right]$

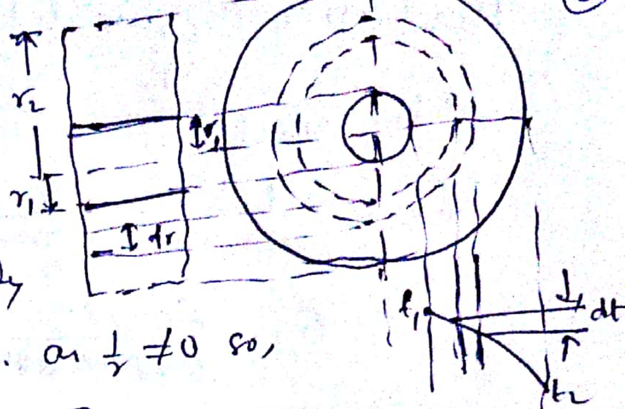
$R_t = \frac{10^{-3}}{144}[300 + 2250 + 133.33] = \frac{10^{-3}}{144} \times 2683.33$

$R_t = 0.0186 \, ^\circ/W$

Heat flow rate $Q = \frac{\Delta t}{R_t} = \frac{24-(-20)}{0.0186} = \frac{44}{0.0186}$

$Q = 2365.6 \, W$

Q.2 General heat conduction eqn in cylinderical coordinates is given by



$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

for steady state conduction $\frac{\partial t}{\partial \tau} = 0$.

If heat flows along radial direction only then

$$\frac{d^2 t}{dr^2} + \frac{1}{r}\frac{dt}{dr} = 0 \quad \text{or,} \quad \frac{1}{r}\frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0. \quad \text{as } \frac{1}{r} \neq 0 \text{ so,}$$

$$\frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0 \quad \text{or} \quad r\frac{dt}{dr} = C_1 \quad \text{—— (1)}$$

$$r\,dt = C_1\,dr \quad \therefore dt = C_1\frac{dr}{r}$$ integrating both sides we get

$$\int_0^t dt = C_1 \int_0^r \frac{dr}{r} \Rightarrow t = C_1 \ln r + C_2 \quad \text{—— (2)}$$

at $r = r_1$, $t = t_1$ & at $r = r_2$, $t = t_2$

$\therefore$ In (2) we get $t_1 = C_1 \ln r_1 + C_2$ — (i) & $t_2 = C_1 \ln r_2 + C_2$ — (ii)

Solving (i) & (ii) we get $C_1 = -\dfrac{(t_1 - t_2)}{\ln\frac{r_2}{r_1}}$ & $C_2 = t_1 + \dfrac{t_1 - t_2}{\ln\frac{r_2}{r_1}} \cdot \ln r_1$

Putting in eqn (2) we get

$$t = t_1 + \frac{t_1 - t_2}{\ln\frac{r_2}{r_1}} \cdot \ln r_1 - \frac{(t_1 - t_2)}{\ln\frac{r_2}{r_1}}\ln r \Rightarrow t - t_1 = \frac{1}{\ln\frac{r_2}{r_1}}\left[(t_1 - t_2)\ln r_1 - (t_1 - t_2)\ln r\right]$$

$$(t - t_1)\ln\frac{r_2}{r_1} = (t_1 - t_2)\ln\frac{r_1}{r} = (t_2 - t_1)\ln\frac{r}{r_1}$$

$$\boxed{\therefore \frac{t - t_1}{t_2 - t_1} = \frac{\ln\frac{r}{r_1}}{\ln\frac{r_2}{r_1}}}$$

OR

There is a decrease in heat transfer rate upto certain extent on increasing insulation thickness but there is an optimal value of thickness of insulation, after which on further increase of insulation ~~conduction~~ or heat transfer rate increase. This optimal value of insulation thickness is called critical radius of insulation for cylinderical or spherical metallic enclosures.

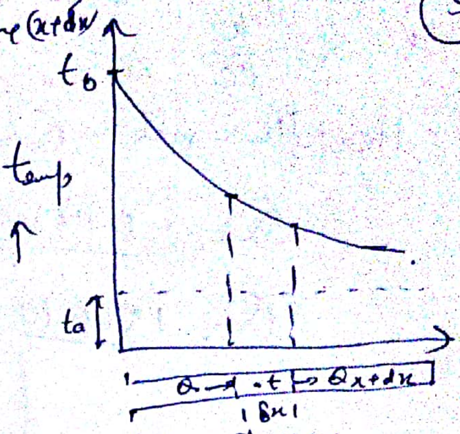
Min insulation thickness or radius $r_c = \frac{k}{h} = \frac{0.18}{8.5} = 0.021118\,m$

$r_c = 0.02118\,mm = 0.0212\,m = 21.2\,mm.$

$\therefore$ Min thickness of insulation $= r_c - r_i = 21.2 - \frac{8}{2} = 17.2\,mm$

OR ~~3(b)~~ Q.3. (5)

Heat conducted out of the element at plane $(x+dx)$



$$Q_{x+dx} = -kA_c\left(\frac{dt}{dx}\right)_{x+dx}$$

$$= -kA_c\frac{d}{dx}\left(t+\frac{dt}{dx}\cdot\delta x\right)$$

Heat convected out of element between planes x and $(x+\delta x)$

$$Q_{conv} = h(P\,dx)(t-t_a)$$

from heat balance $\quad Q_x = Q_{x+\delta x} + Q_{conv}$. $\quad$ k-x → ~~thickness~~. → ~~thickness~~ length

$$-kA_c\left(\frac{dt}{dx}\right) = -kA_c\frac{d}{dx}\left(t+\frac{dt}{dx}\cdot\delta x\right) + h\cdot(P\cdot\delta x)(t-t_a)$$

$$-kA_c\left(\frac{dt}{dx}\right) = -kA_c\frac{d}{dx}\left(t+\frac{dt}{dx}\cdot\delta x\right) - h(P\,dx)(t-t_a) = 0.$$

$$\cancel{-kA_c\frac{dt}{dx}} + \cancel{kA_c\frac{dt}{dx}} + kA_c\frac{d^2t}{dx^2}\delta x = h(P\,\delta x)(t-t_a) \Rightarrow \frac{d^2t}{dx^2} = \frac{hP}{kA_c}(t-t_a)$$

$$kA_c\frac{d^2t}{dx^2}\cancel{\delta x} = h(P\,\cancel{\delta x})(t-t_a)$$

$$\frac{d^2t}{dx^2} - \frac{hP}{kA_c}(t-t_a) = 0 \quad \text{— (1)}$$

If $\theta = t - t_a$ then $\dfrac{d\theta}{dx} = \dfrac{dt}{dx}$ & $\dfrac{d^2\theta}{dx^2} = \dfrac{d^2t}{dx^2}$

$\therefore$ (1) may be written as. $\quad \dfrac{d^2\theta}{dx^2} - \dfrac{hP}{kA_c}\theta = 0$

If $m = \sqrt{\dfrac{hP}{kA_c}}$ then $\quad \dfrac{d^2\theta}{dx^2} - m^2\theta = 0 \quad$ — (2)

the general soln of this eqn (2) is

$$\theta = C_1e^{mx} + C_2e^{-mx} \quad \text{— (3)}$$

For evaluating C_1 & C_2 boundary conditions are used

~~first~~ for infinitely long fin.

First boundary condition is $\theta = \theta_0$ when $x = 0$.

$\therefore$ ~~from~~ eqn (3) $\quad \theta_0 = C_1 + C_2$ — (a)

Second boundary condition is $l \to \infty$ i.e. $x = \infty$. $\theta = 0$.

$\therefore$ from eqn (3) $\quad 0 = C_1e^{m\infty} + C_2e^{-m\infty} \Rightarrow C_1e^{m\infty} + 0 = 0$

$\therefore C_1 = 0$. $\quad$ Putting in (a) $\quad C_2 = \theta_0$

Putting in (3) these values of C_1 & C_2

$$\theta = \theta_0e^{-mx} \quad \text{i.e} \quad t - t_a = (t_0 - t_a)e^{-mx} \quad \text{— (4)}$$

An estimation of heat flow rate can be made from conduction eqn.

$$Q_{fin} = -kA_c\left(\frac{dt}{dx}\right)_{x=0}$$

from eqn (4) $\quad t - t_a = (t_0 - t_a)e^{-mx}$

i.e. $t = t_a + (t_0 - t_a)e^{-mx}$

$\therefore \dfrac{dt}{dx} = \left[0 + (-m)(t_0 - t_a)e^{-mx}\right]_{x=0}$

$\dfrac{dt}{dx} = -m(t_0 - t_a) \quad \therefore Q_{fin} = kA_c m(t_0 - t_a)$

$= (\sqrt{PhkA_c})(t_0 - t_a)$

Soln 4 Thermal Diffusivity is denoted by α it is the ratio of conductivity of material (k) to the product of its density and specific heat capacity. ⑥

i.e $\alpha = \frac{k}{\rho C}$.

Heat flow rate $Q = -kA\frac{dt}{dx} = -0.116 \times 0.5 \times \frac{30-300}{10 \times 10^{-3}}$

$Q = -0.116 \times 0.5 \times (-270) \times 10^2 = 0.116 \times 0.5 \times 270 \times 100$ W

$Q = 1566$ W.

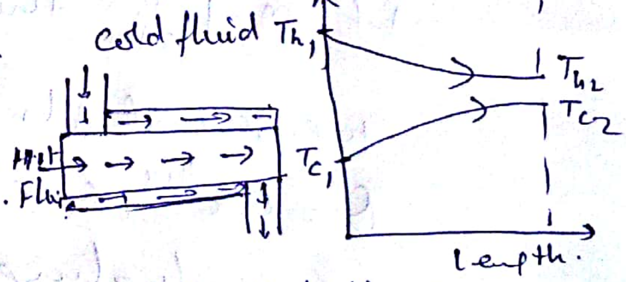
OR.

Heat exchanger is a device through which the heat transfer takes place from one fluid to the other. Heat exchanger transfer heat energy, this transfer may occur to a single fluid or between two fluids subject to the condition that the two fluids are at different temperatures.

According to relative direction of fluid flow the heat exchanges are of two types.

① Parallel flow heat exchangers.

In this type of heat exchangers the fluids flow in same direction.



Parallel flow H.E.

② Counter flow heat exchanger

In this type of Heat Exchanger the direction of fluid flow is opposite to one & other.