Page (1) Solution of SET-B. (HEAT TRANSFER) ME-V sem

Soln. 1 (a) There are three modes of heat transfer:- Conduction, Convection and Radiation.

Conduction:- In conduction the heat is transferred from a region of higher temperature to a region of low temperature within a solid medium or between mediums in direct contact.

The rate equation of one dimensional steady flow by Conduction is prescribed by Fourier Law according to which "the rate of heat conduction is directly proportional the surface area perpendi- -cular to the direction of heat flow and temperature gradient along the direction of heat flow." i.e. $Q \propto A\left(\frac{dt}{dx}\right)$

$\therefore Q = -KA\frac{dt}{dx}$ Here K is conductivity of material.

Convection:- It is a process of energy transport affected by the circulation or mixing of a fluid medium. Convection is possible only in a fluid medium and is directly linked with the transport of medium itself.

The rate equation for convective heat transfer between a surface and adjacent fluid is given by Newton's Law of Cooling. According to which "The rate of heat transfer is directly proportional to the product of Area of surface and difference of temperature of surface and fluid."

$$Q \propto A.(t_s - t_f) \Rightarrow Q = hA(t_s - t_f)$$

Here h is convective coefficient of medium fluid.

Radiation:- During this method of heat transfer material medium is not required between source and receiver. Radiation is more effective in vacuum. Presence of material reduces the heat transfer rate.

The rate equation of Radiation heat transfer is given by Stefan-Boltzman Law. According to which "Emissivity of surface is directly proportional to the product of surface area and 4th power of its emitting absolute temperature"

i.e $E_b \propto AT^4$ $E_b = \sigma_b AT^4$.

where $\sigma_b = 5.67 \times 10^{-8}\ W/m^2K^4$ this is Stefan Boltzman Constant.

Soln 1(b) Temperature Gradient $\frac{dT}{dx} = \frac{t_2 - t_1}{\delta} = \frac{32-106}{12\times10^{-2}} = -\frac{74\times10^2}{12}$

Temp Gradient $= -6.167\times10^2$ °C/m.

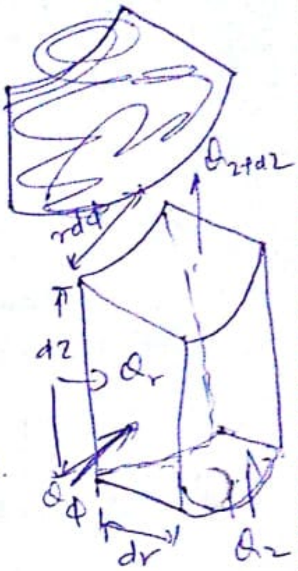
Heat conducted $= Q = -kA\frac{dt}{dx} = -8.2\times3\times(-6.167\times10^2) = \underline{15170.82\,W}$

OR

Heat conduction Eqn in Polar coordinate system

In this case elemental vol $= (dr.rd\phi.dz)$

Area of faces are $(rd\phi.dz)$ $(dr.dz)$ & $(rd\phi.dr)$



Heat influx in radial direction

$$Q_r = -k(rd\phi\, dz)\frac{\partial t}{\partial r}d\tau$$

Heat efflux $Q_{r+dr} = Q_r + \frac{\partial}{\partial r}(Q_r)dr$

$$\therefore dQ_r = Q_r - [Q_{r+dr}] = -\frac{\partial}{\partial r}(Q_r)dr$$

$$\therefore dQ_r = k(dr.d\phi\, dz)\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)d\tau = k(dr\,d\phi\,dz)\left[r\frac{\partial^2 t}{\partial r^2} + \frac{\partial t}{\partial r}\right]d\tau$$

$$\therefore dQ_r = k(dr.\,rd\phi\,dz)\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right]d\tau \quad —(1)$$

Heat influx in Tangential Direction $Q_\phi = -k(dr.dz)\frac{\partial t}{r\partial\phi}d\tau$

Heat ~~stored~~ efflux $Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{r\partial\phi}(Q_\phi)r.d\phi$

Heat stored in element due to tangential direction

$$dQ_\phi = Q_\phi - Q_{\phi+d\phi} = -\frac{\partial}{r\partial\phi}(Q_\phi)r.d\phi$$

$$dQ_\phi = \frac{\partial}{r\partial\phi}\left\{-k(dr.dz)\frac{\partial t}{r\partial\phi}d\tau\right\}r\,d\phi = (k\,dr.dz\,d\phi)\left[\frac{1}{r}\frac{\partial^2 t}{\partial\phi^2}\right]d\tau$$

$$dQ_\phi = (k.dr.dz.rd\phi)\left[\frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2}\right]d\tau \quad — (2)$$

In Axial direction Heat influx $Q_z = -k(dr.rd\phi)\frac{\partial t}{\partial z}d\tau$

Heat efflux $Q_{z+dz} = Q_z + \frac{\partial}{\partial z}(Q_z)dz$

Heat Stored $dQ_z = Q_z - Q_{z+dz} = -\frac{\partial}{\partial z}(Q_z)dz = k(dr.rd\phi.dz)\frac{\partial^2 t}{\partial z^2}.d\tau$ —(3)

Heat generation within control volume $= mc\,dt = \rho(dv)c\frac{\partial t}{\partial\tau}.d\tau$ —(4)

From Heat balance

$$dQ_r + dQ_\phi + dQ_z + q_g(dv)d\tau = \rho(dv)c\frac{\partial t}{\partial\tau}.d\tau$$

$$k(dr.rd\phi\,dz)\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right]\cancel{d\tau} + k(dr.dz.rd\phi)\left[\frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2}\right]\cancel{d\tau} + k(\cancel{dr.rd\phi.dz})\frac{\partial^2 t}{\partial z^2}.\cancel{d\tau} + q_g(dv)\cancel{d\tau} = \rho\cancel{(dv)}c\frac{\partial t}{\partial\tau}.\cancel{d\tau}$$

$$k\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right] + k\left[\frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2}\right] + k\left[\frac{\partial^2 t}{\partial z^2}\right] + q_g = \rho c\frac{\partial t}{\partial\tau}$$

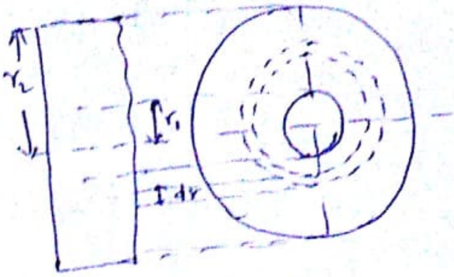
So $$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho c}{k}\frac{\partial t}{\partial\tau}$$

$$\boxed{\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial\tau}}$$ This is reqd. eqn.

Soln 2

Heat conduction eqn in polar coordinates is

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$



For steady state $\frac{\partial t}{\partial \tau} = 0$ again in one dimensional heat flow i.e. heat flow along the radial direction only.

$$\frac{d^2 t}{dr^2} + \frac{1}{r}\frac{dt}{dr} = 0 \Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0$$

i.e. $\frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0$, $\therefore r\frac{dt}{dr} = C_1$

$\therefore r\frac{dt}{dr} = C_1$ — (1) $\Rightarrow r\,dt = C_1\,dr$

~~integrating we get~~ $\therefore dt = \frac{C_1}{r}dr$..

integrating we get $\int_0^t dt = C_1 \int_0^r \frac{dr}{r}$

$t = C_1 \ln r + C_2$ — (2)

at $r = r_1$, $t = t_1$ & at $r = r_2$ $t = t_2$ putting in (2) we get

$$C_1 = -\frac{t_1 - t_2}{\ln\frac{r_2}{r_1}} \quad \& \quad C_2 = t_1 + \frac{t_1 - t_2}{\ln\frac{r_2}{r_1}}\ln r_1$$

$\therefore$ (2) reduces to

$$t = t_1 + \frac{t_1 - t_2}{\ln\frac{r_2}{r_1}}\ln r_1 - \frac{t_1 - t_2}{\ln\frac{r_2}{r_1}}\ln r \quad \text{— (3)}$$

$$(t - t_1)\ln\frac{r_2}{r_1} = (t_2 - t_1)\ln\frac{r}{r_1}$$

$$\boxed{\therefore \frac{t - t_1}{t_2 - t_1} = \frac{\ln\frac{r}{r_1}}{\ln\frac{r_2}{r_1}}} \quad \text{— (4)}$$

Again $Q = -kA\frac{dt}{dr}$

$$Q = -k(2\pi r l)\frac{d}{dr}\Big[\underbrace{t_1 + \frac{t_1 - t_2}{\ln\frac{r_2}{r_1}}\ln r_1}_{\text{Const term}} - \frac{(t_1 - t_2)}{\ln\frac{r_2}{r_1}}\ln r\Big]$$

$$Q = k(2\pi r l)\left(\frac{t_1 - t_2}{\ln\frac{r_2}{r_1}}\right)\cdot\frac{1}{r}$$

$$Q = 2\pi k l\left(\frac{t_1 - t_2}{\ln\frac{r_2}{r_1}}\right)$$

$\therefore$ ~~$\frac{t_1 - t_2}{Q} = R_t$~~

as $\frac{\Delta t}{Q} = R_t$.

$$\therefore \frac{t_1 - t_2}{Q} = \frac{\ln\frac{r_2}{r_1}}{2\pi k l}.$$

$$\boxed{\therefore R_t = \frac{\ln\frac{r_2}{r_1}}{2\pi k l}}$$

OR

2 (i) During steady state of heat transfer the temp. diff. of a place is independent of time i.e. $\frac{dt}{d\tau} = 0$. where as during ~~unstead~~ unsteady heat flow the change in temperature is dependent on time mathematically during unsteady flow of heat $\frac{dt}{d\tau} \neq 0$

$R_{t_1} = \frac{\delta_1}{k_1 A}$, $R_{t_2} = 0.16$

$R_{t_3} = \frac{\delta_3}{k_3 A}$, $R_{t_4} = \frac{\delta_4}{k_4 A}$

$$R_t = \frac{1}{A}\left[\frac{\delta_1}{k_1} + \frac{\delta_2}{k_2} + \frac{\delta_3}{k_3}\right] + 0.$$

$$R_t = \frac{1}{1}\left[\frac{0.125}{1.6} + \frac{0.15}{0.3} + \frac{0.012}{0.14}\right] + 0.16$$

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$R_t = 0.0781 + 0.5 + 0.0857 + 0.16$

$R_t = 0.8238\ ^\circ C/W$

Convective heat resistance $= \frac{1}{h_o A} = \frac{1}{17\times 1} = 0.059$

$\therefore R_{total} = 0.8238 + 0.059 = 0.8828^\circ C/W$

$$Q = \frac{\Delta t}{R_{total}} = \frac{1100-25}{0.8828} = 1217.72\ W/m^2$$

$Q = 1.2177\ kW/m^2$

Let to is the temperature of outside surface of wall then

$Q = h_o A(t_o - t_a)$

$1217.72 = 17\times 1(t_o - 25) \quad \therefore t_o = 96.63^\circ C$

Q3(a) Heat conduction is given by

$$Q = -kA \cdot \frac{dt}{dx}$$

If wall is δ thick then $dx = \delta$.

$$\therefore Q = -kA\frac{dt}{\delta}$$

~~New Resistance~~

Now Thermal Resistance $= R_t = \frac{\Delta t}{Q}$

$$R_t = \frac{dt}{Q} = \frac{\delta}{KA}$$

$$\boxed{R_t = \frac{\delta}{KA}}$$

3(b) The floor is not considered to conduct the heat.

$\therefore$ Area for heat flow,

$A = 2[(10\times 2.5) + (8\times 2.5)] + (10\times 8)$

$= 90 + 80 = 170\ m^2$

Thermal Resistance R_t

$$R_t = \frac{1}{A}\left[\frac{\delta_1}{k_1} + \frac{\delta_2}{k_2} + \frac{\delta_3}{k_3}\right] = \frac{1}{170}\left[\frac{60\times 10^{-3}}{0.2} + \frac{90\times 10^{-3}}{0.04} + \frac{240\times 10^{-3}}{1.8}\right]$$

$$R_t = \frac{10^{-3}}{170}[300 + 2250 + 133.33] = 0.01578^\circ C/W$$

$$Q = \frac{\Delta t}{R_t} = \frac{25-(-20)}{0.01578} = \frac{45}{0.01578} = 2851.7\ W$$

3 OR (3)

Sol (2)

Temperature at the end of fin of an infinite length becomes equal to surrounding temperature so boundary conditions are

$t = t_0$ at $x = 0$.

$\therefore \theta = t - t_a = t_0 - t_a = \theta_0$

i.e. at $x=0$, $t = t_0$ & $\theta = \theta_0$

at $x = \infty$, $t = t_a \quad \therefore \theta = t - t_a$

$\therefore \theta = t_a - t_a = 0$.

i.e at $x = \infty$, $t = t_a$ & $\theta = 0$

from eqn. of temp. for fins.

$\theta = C_1 e^{mx} + C_2 e^{-mx}$ — (1)

$\therefore$ at $x = 0$.

$\theta_0 = C_1 e^0 + C_2 e^{-0} = C_1 + C_2$

i.e. $C_1 + C_2 = \theta_0$ — (a)

at $x = \infty$

$0 = C_1 e^{m\infty} + C_2 e^{-m\infty}$

i.e $C_1 e^{m\infty} + C_2 e^{-m\infty} = 0$ — (b)

Again as $e^{-m\infty} = 0$. so

$C_1 e^{m\infty} = 0 \quad$ i.e $C_1 = 0$.

Putting in (b) we get $C_2 = \theta_0$

Putting in (1) we get $\theta = \theta_0 e^{-mx}$

$t - t_a = (t_0 - t_a)e^{-mx}$ — (2)

$Q_{fin} = -kA_c\left(\frac{dt}{dx}\right)_{x=0}$

$\frac{dt}{dx} = \frac{d}{dx}\left[t_a + (t_0 - t_a)e^{-mx}\right]$

$\left(\frac{dt}{dx}\right)_{x=0} = -m(t_0 - t_a)$

$\therefore Q_{fin} = -kA_c\left(\frac{dt}{dx}\right)_{x=0}$

$= -kA_c[-m(t_0 - t_a)]$

$Q_{fin} = mkA_c(t_0 - t_a)$

Again $m = \sqrt{\frac{hP}{kA_c}}$

$\therefore Q_{fin} = \left(\sqrt{\frac{hP}{kA_c}}\right)kA_c(t_0 - t_a)$

$$Q_{fin} = \left(\sqrt{PhkA_c}\right)(t_a - t_0)$$

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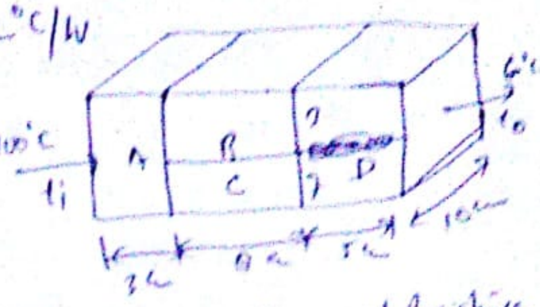
Soln 4

$R_a = \frac{\delta_a}{k_a A_a} = \frac{0.03}{150\times(0.1\times0.1)} = 0.02\,^\circ C/W$

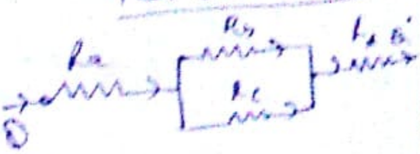


$R_b = \frac{\delta_b}{k_b A_b} = \frac{0.08}{30\times(0.1\times0.03)} = 0.89\,^\circ C/W$

$R_c = \frac{\delta_c}{k_c A_c} = \frac{0.08}{65\times(0.1\times0.07)} = 0.176\,^\circ C/W$

$R_d = \frac{\delta_d}{k_d A_d} = \frac{0.05}{50(0.1\times0.1)} = 0.01\,^\circ C/W$

Thermal Resistance



Total thermal Resistance

$$R_t = R_a + \frac{1}{\left(\frac{1}{R_b}+\frac{1}{R_c}\right)} + R_d = 0.02 + \left(\frac{1}{\frac{1}{0.89}+\frac{1}{0.176}}\right) + 0.01$$

$$= 0.177\,^\circ C/W$$

Heat transfer from wall $= Q = \frac{\Delta t}{R_t}$

$$Q = \frac{t_i - t_o}{R_t} = \frac{400-60}{0.177} = 1921.99 \text{ watt} \simeq 1922 \text{ watt}$$

OR

Soln 4

Laws of Heat Transfer

(a) For conduction of heat according to Fourier law.

"The Rate of Heat Conduction is directly proportional to the surface area perpendicular to the direction of heat flow and it is also directly proportional to the temperature gradient $\left(\frac{dt}{dx}\right)$

Mathematically.

$\cancel{Q = -KA}$ $Q \propto A$ & $Q \propto \frac{dt}{dx}$.

Combining we get $Q \propto A\left(\frac{dt}{dx}\right)$

$Q = -kA\left(\frac{dt}{dx}\right)$ where K is conductivity of material and is fixed for a material it is the property of material.