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RIET

I - Mid Term Paper Solution III Semester-2018-19
Subject - MOS (3ME1A)
Mechanical Engineering
Faculty - Jogendra choudhary

Set-A

Sec-A

1. Define stress, strain & Elasticity.

Ans The Resistance per unit area to deformation is Called stress. The deformation per unit length is termed as strain. The property of Certain materials of returning back to their original position after removing the external force, is Called elasticity.

2. State clearly the Hooke's Law?

Ans When a material is loaded within its elastic limit, the stress is proportional to the stress.

3. Explain the difference between 'primary strain' and 'Secondary strain'?

Ans The deformation of the bar per unit length in the direction of the force is known as primary or linear strain.
The deformation of the bar per unit length in the direction Normal to the force is known as secondary strain.

4 Define the term shear force and Bending moment.

Ans The shear force at the Cross-section of the Beam, is defined as the unbalanced Vertical force to the Right or left of the section.

The Bending moment at the Cross- section of a Beam. is defined as the Unbalanced Vertical force to the right or left of the section.

5. Define the term Bending stress?

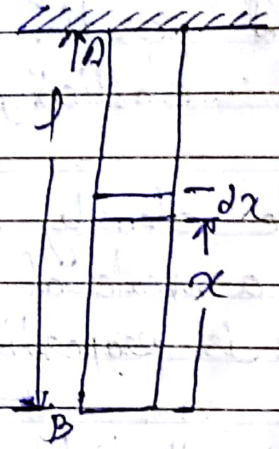
Ans. The resistance offered by the internal stresses, to the Bending, is Called Bending stress.

Sec-B

Q.1. Derive from fundamental, the relation for the deformation of a body subjected to its own weight.

A.

$P = wAx$



Elongation of the small section of the bar, due to weight of the bar for a small section of length x

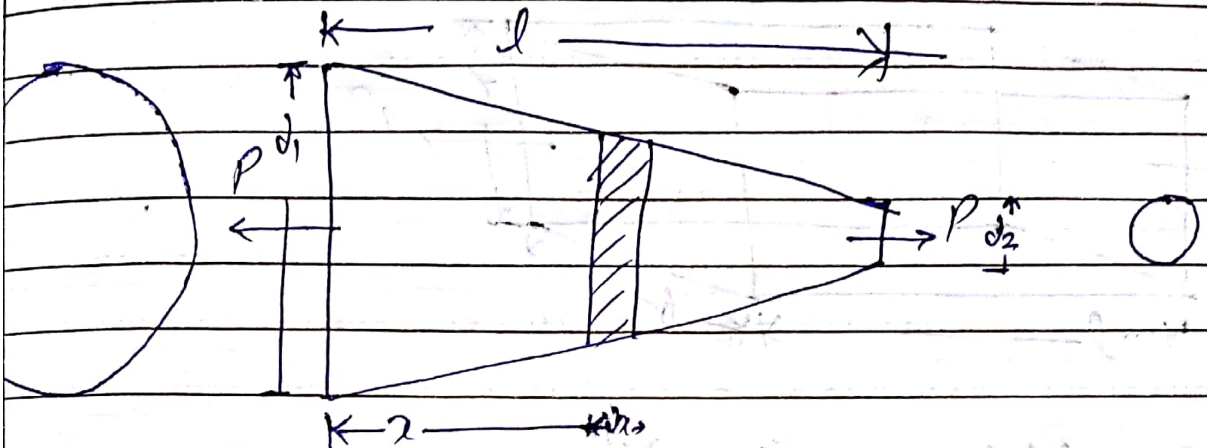
$$= \frac{Pl}{AE} = \frac{(wAx).dx}{AE} = \frac{w.x dx}{E}$$

$$\delta l = \int_0^l \frac{w.x.dx}{E}$$

$$\delta l = \frac{w}{E}\left[\frac{x^2}{2}\right]_0^l = \frac{wl^2}{2E}$$

$$\boxed{\delta l = \frac{wl^2}{2E} = \frac{Wl}{2AE}}$$

Obtain a relation for the elongation of a uniformly tapering circular bar.



$$d_x = d_1 - \frac{(d_1-d_2)}{l}x = d_1 - K\cdot x \qquad \left\{ K = \frac{d_1-d_2}{l} \right\}$$

$$A_x = \frac{\pi}{4}(d_1-Kx)^2$$

$$\sigma_x = \frac{P}{\frac{\pi}{4}(d_1-Kx)^2} = \frac{4P}{\pi(d_1-Kx)^2}$$

$$\delta l_x = \frac{4\cdot P\cdot dx}{\pi(d_1-Kx)^2 E}$$

$$\delta l = \int_0^l \frac{4P\cdot dx}{\pi(d_1-Kx)^2 E}$$

$$\delta l = \frac{4P}{\pi E K}\left[\frac{1}{d_1-Kl} - \frac{1}{d_1}\right]$$

put $K = \frac{d_1-d_2}{l}$

$$\boxed{\delta l = \frac{4Pl}{\pi E d_1 d_2}}$$

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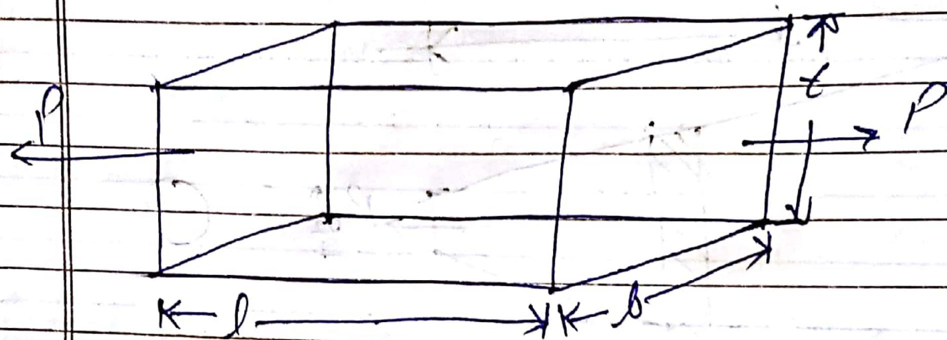
3. Derive a Relation for the Volumetric strain of a body.

Ans.



$V = lbt$

$V + \delta V = (l + \delta l)(b - \delta b)(t - \delta t)$

$\delta V = V + \delta V - V = lbt\left[1 + \frac{\delta l}{l} - \frac{\delta b}{b} - \frac{\delta t}{t}\right] - lbt$

$\delta V = lbt\left[\frac{\delta l}{l} - \frac{\delta b}{b} - \frac{\delta t}{t}\right]$

$\delta V = V \cdot \frac{P}{btE}\left(1 - \frac{2}{m}\right)$ $\left\{\frac{\frac{\delta l}{l}}{\frac{\delta b}{b}} = \frac{1}{m}\right\}$

$\frac{\delta V}{V} = \frac{V \times \frac{P}{btE}\left(1 - \frac{2}{m}\right)}{V}$

$\frac{\delta V}{V} = \frac{P}{btE}\left(1 - \frac{2}{m}\right)$

$\boxed{\delta V = \varepsilon\left(1 - \frac{2}{m}\right)}$

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4. Prove $E = 3K\left(1-\frac{2}{m}\right)$

A. When a body is subjected to three mutually perpendicular stresses of equal intensity, the ratio of direct stress to the corresponding strain is known as Bulk modulus

$$K = \frac{\sigma}{\delta V/V}$$

$$\frac{\delta V}{V} = \frac{3\sigma}{E}\left(1-\frac{2}{m}\right)$$ {as from previous question}

$$K = \frac{\sigma}{\frac{3\sigma}{E}\left(1-\frac{2}{m}\right)}$$

$$K = \frac{mE}{3(m-2)}$$

$$\boxed{E = 3K\left(1-\frac{2}{m}\right)}$$

Q.5 What is the value of modulus of rigidity of a steel alloy, if the modulus of elasticity is 180 GPa and Poisson's ratio is 0.25?

A. Given $E = 180\text{ GPa}$

$\frac{1}{m} = 0.25 \Rightarrow m = 4$

We know that $C = \frac{mE}{2(m+1)}$

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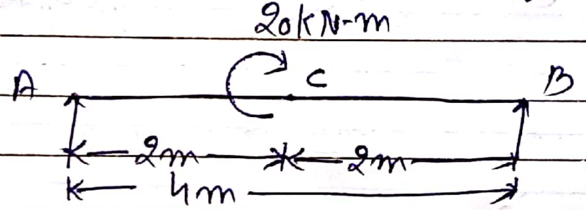
$C = \dfrac{4 \times 180}{2(4+1)}$

$C = \dfrac{4 \times 180}{2 \times 5}$

$\boxed{C = 72\ GPa}$ Ans.

Q.6. A Simply supported Beam AB of span 4m is subjected to a clockwise moment of 20 kN-m at its Center. Draw the S.F. and B.M. diagram.

Ans.



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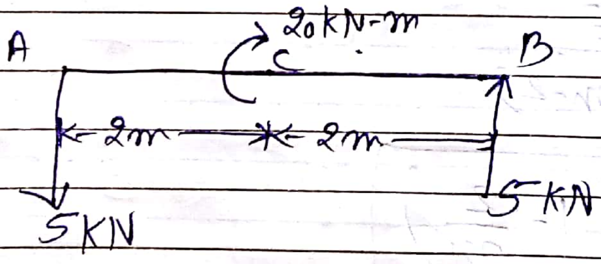
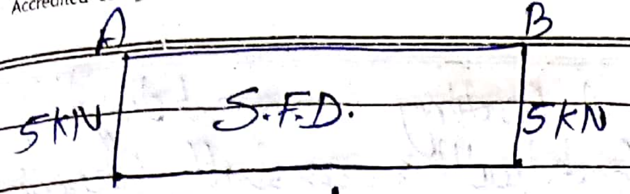
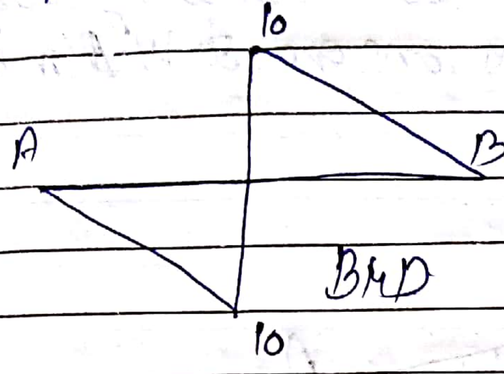


B-C	$F_x = -5$	$M_x = 5.x$
$x=0$	$F_B = -5$ KN	$M_B = 0$
$x=2$	$F_C = -5$ KN	$M_C = 10$ KN-m
C-A	$F_x = -5$	$M_x = 5.x - 20$
$x=2$	$F_C = -5$ KN	$M_C = -10$ KN-m
$x=4$	$F_A = -5$ KN	$M_A = 0$

$-R_{13} \times 4 + 20$
5

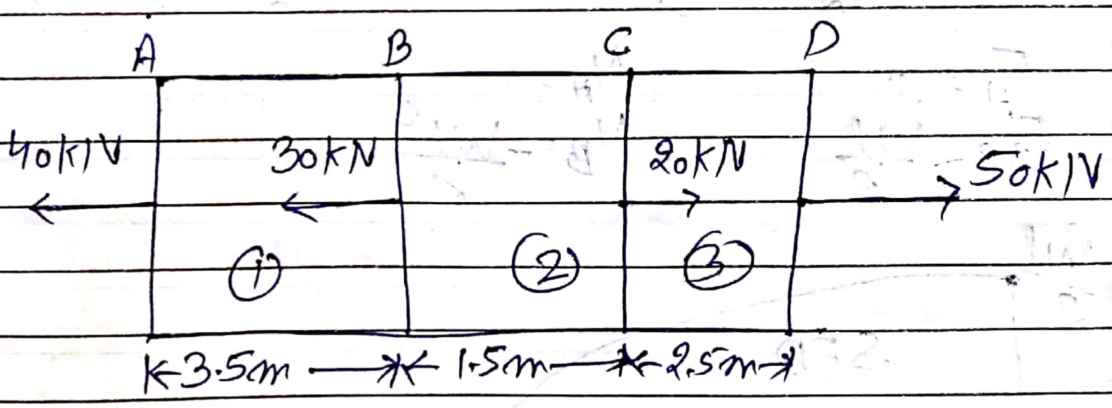




Sec - C.

Q.1. A Copper rod ABCD of 800 mm^2 Cross-section area and 7.5 m long is subjected to force as shown in figure. Find the elongation of the bar if E = 100 GPa

A.



$\delta l = \delta l_1 + \delta l_2 + \delta l_3$

$$\delta l = \frac{P_1 \cdot l_1}{A_1 \cdot E} + \frac{P_2 l_2}{A_2 E} + \frac{P_3 l_3}{A_3 E}$$

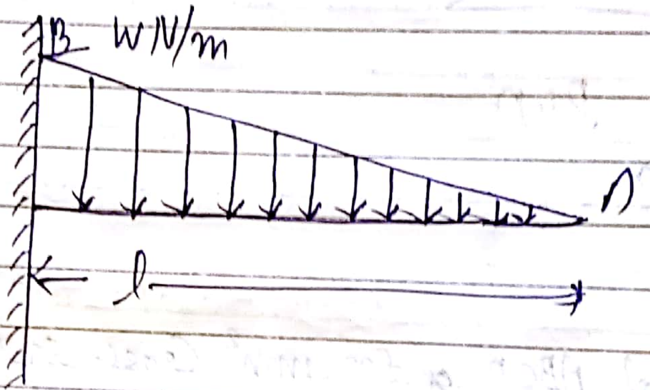
$$\delta l = \frac{40\times10^3 \times 3.5}{800\times10^{-6}\times100\times10^9} + \frac{70\times10^3\times1.5}{800\times10^{-6}\times100\times10^9} + \frac{50\times10^3\times2.5}{800\times10^{-6}\times100\times10^9}$$

$$\delta l = \frac{4\times3.5}{8\times10^3} + \frac{7\times1.5}{8\times10^3} + \frac{5\times2.5}{8\times10^3}$$

$\delta l = 4.6\times10^{-3}$ m $\Rightarrow$ $\boxed{\delta l = 4.6 \text{ mm}}$ Ans.

Q.2. Draw the shear force and Bending moment diagram for a Cantilever Beam loaded with a gradually Varying Load of intensity zero from one end to W N/m at the other end.

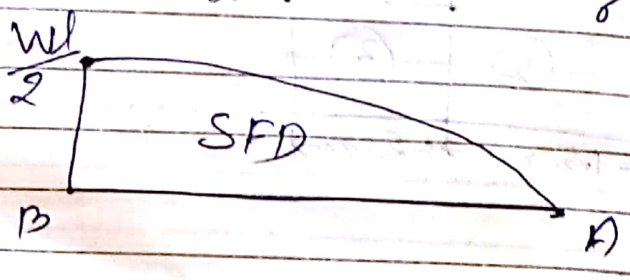
A.

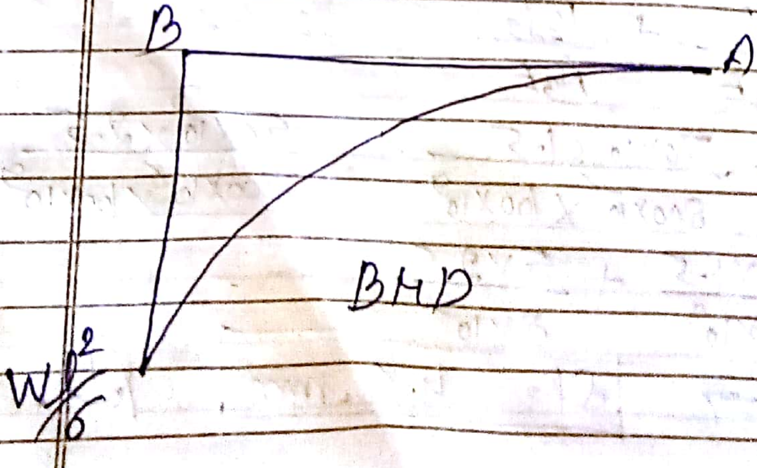


A-B $\quad F_x = \frac{1}{2} \cdot x \cdot \frac{Wx}{l} \qquad M_x = -\frac{1}{2} \cdot x \cdot \frac{Wx}{l} \cdot \frac{x}{3}$

$x = 0 \quad F_A = 0 \qquad M_A = 0$

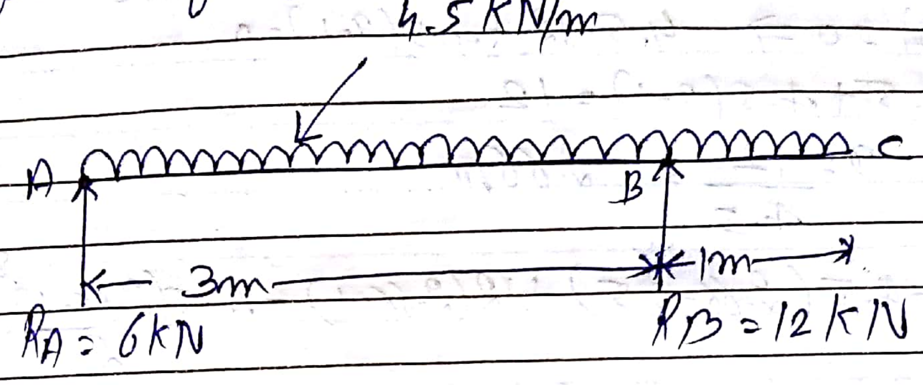
$x = l \quad F_B = \frac{Wl}{2} \qquad M_B = -\frac{Wl^2}{6}$





Q.3 An overhanging Beam is shown in figure. Draw the shear force & Bending Moment diagram and find the point of Contraflexure if any.

A.



$R_A = 6KN$
$R_B = 12KN$

C-B $\quad F_x = 4.5x \qquad M_x = -4.5x \cdot \frac{x}{2}$

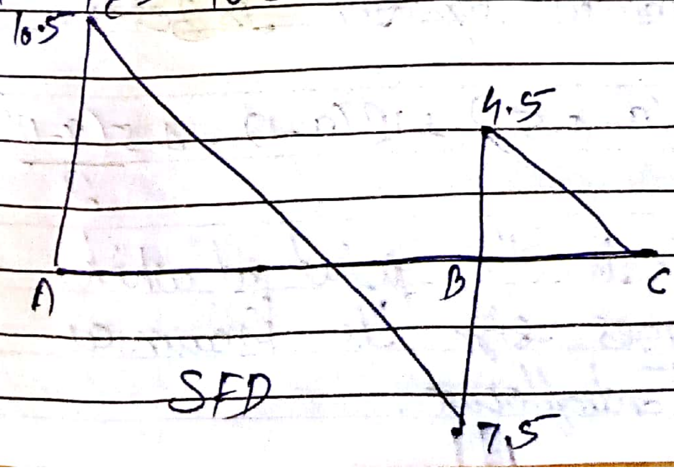
$x=0 \quad F_C = 0 \qquad M_C = 0$

$x=1 \quad F_B = 4.5KN \qquad M_B = -2.25 \ KN\text{-}m$

B-A $\quad F_x = 4.5\times1 - 12 + 4.5(x-1)$; $M_x = -4.5(x-0.5) + 12(x-1) - 4.5(x-1)\frac{(x-1)}{2}$

$x=1 \quad F_B = -7.5KN \qquad M_B = -2.25 \ KN$

$x=4 \quad F_C = 10.5 \ KN \qquad M_C = 0$



SFD

The point at which shear force changes sign the Bending moment at this point is max^m.

$F_x = B-A = 0 \Rightarrow 4.5 - 12 + 4.5(x-1) = 0$

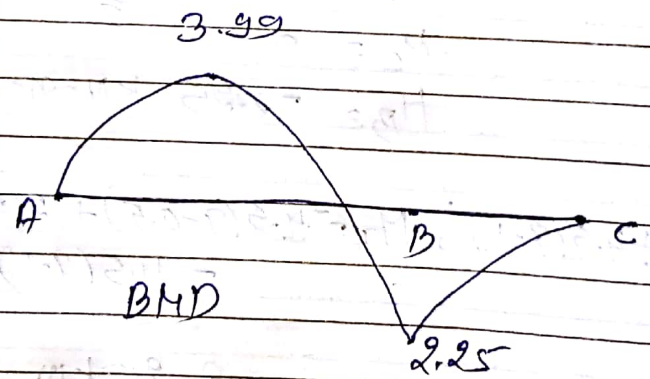
$4.5 + 4.5(x-1) = 12$

$x = \frac{12}{4.5} = 2.66\,m$

$M_{2.66} = -4.5(2.66-0.5) + 12(2.66-1) - 4.5\frac{(2.66-1)^2}{2}$

$M_{2.66} = -9.72 + 19.92 - 6.2001$

$M_{2.66} = 3.99\ kN\text{-}m$



Bending moment is zero in between B-A so that Bending moment equation is zero.

$M_{B-A} = 0 \Rightarrow -4.5(x-0.5) + 12(x-1) - 4.5\frac{(x-1)^2}{2}$

$x = 2.67\ m$ is the point at which Bending moment changes sign is known as point of Contraflexure.

Set - B.

Sec A

1. Derive a relation between stress and strain of an elastic body.

A. $\sigma = \frac{P}{A}$

$\frac{\delta l}{l} = \varepsilon$

$\sigma \propto \varepsilon$

$\sigma = E\varepsilon$

$\frac{P}{A} = E \cdot \frac{\delta l}{l} \Rightarrow \delta l = \frac{Pl}{AE}$

2. Define thermal stress and thermal strain.

A. If the body is allowed to expand or contract and the deformation of the body is prevented, the stress induced is called thermal stress. The corresponding strain are called thermal strain.

3. Define Poisson's Ratio.

A. Within in elastic limit the ratio of lateral strain to linear strain is termed as Possion's Ratio.

4. Explain clearly the term modulus of Rigidity.

A. It has been experimentally found that within the elastic limit, the shear stress is proportional to shear strain.

$\tau \propto \phi$

$\tau = C \cdot \phi$

↳ Modulus of Rigidity

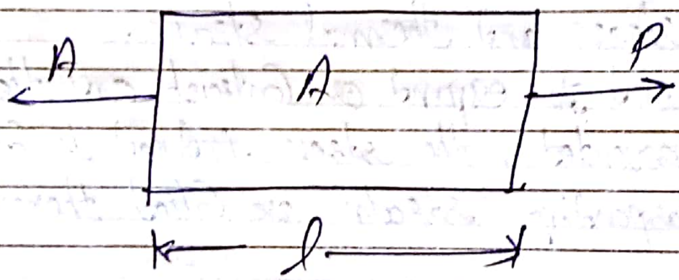
5. What do you understand by the term 'point of Contraflexure.

A. In an over-hanging Beam the point at which Bending Moment changes sign is known as point of Contraflexure.

Sec B

1. Derive from fundamental, the Relation for the deformation of a body subjected to tensile force.

A.



$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{\delta l}{l}$$

$$\sigma \propto \varepsilon$$

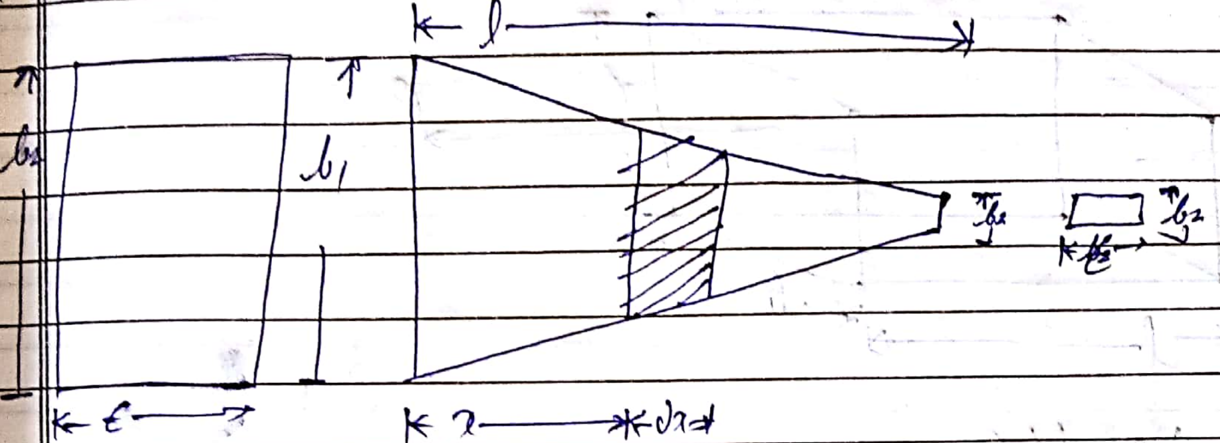
$$\sigma = E\varepsilon$$

$$\frac{P}{A} = E \cdot \frac{\delta l}{l}$$

$$\boxed{\delta l = \frac{P \cdot l}{AE}}$$

Q. Obtain a Relation for the elongation of a Uniformly tapering Rectangular Section.

A.



$$b_x = b_1 - (b_1 - b_2)\cdot\frac{x}{l} = b_1 - k\cdot x \qquad \left\{ k = \frac{b_1 - b_2}{l} \right)$$

$$A_x = b_x \cdot t = (b_1 - kx)\cdot t$$

$$\delta l_x = \frac{P\cdot dx}{(b_1 - kx)\, tE}$$

$$\delta l = \int_0^l \frac{P\cdot dx}{(b_1 - kx)\, tE}$$

$$\delta l = \frac{P}{tE}\Big[\log(b_1 - kx)\Big]_0^l$$

$$\boxed{\delta l = \frac{Pl}{tE(b_1 - b_2)} \log_e\left(\frac{b_1}{b_2}\right)}$$

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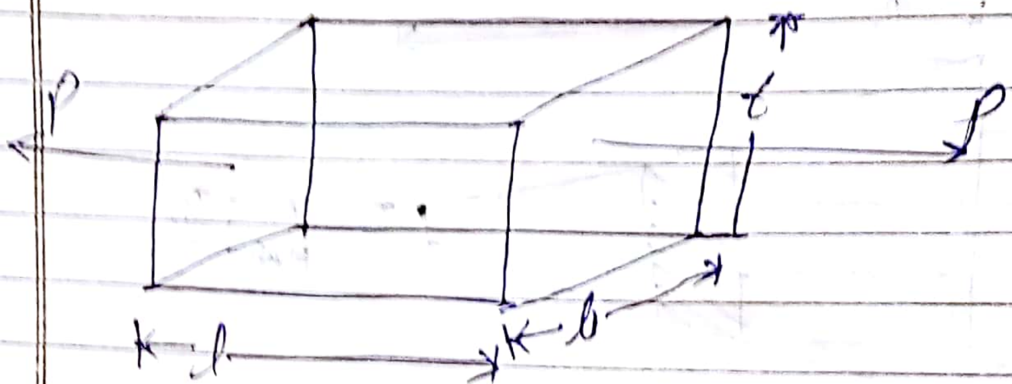
3\. Derive a Relation for the Volumetric Strain for a body.

A.



$V = lbt$

$V + \delta V = (l + \delta l)(b - \delta b)(t - \delta t)$

$\delta V = V + \delta V - V = lbt\left[1 + \frac{\delta l}{l} - \frac{\delta b}{b} - \frac{\delta t}{t}\right] - lbt$

$\delta V = lbt\left[\frac{\delta l}{l} - \frac{\delta b}{b} - \frac{\delta t}{t}\right]$

$\delta V = \frac{V \cdot P}{btE}\left(1 - \frac{2}{m}\right)$

$\frac{\delta V}{V} = \frac{V \times P}{btE}\left(1 - \frac{2}{m}\right)$

$\delta V = \frac{P}{btE}\left(1 - \frac{2}{m}\right)$

$\boxed{\delta V = \varepsilon\left(1 - \frac{2}{m}\right)}$

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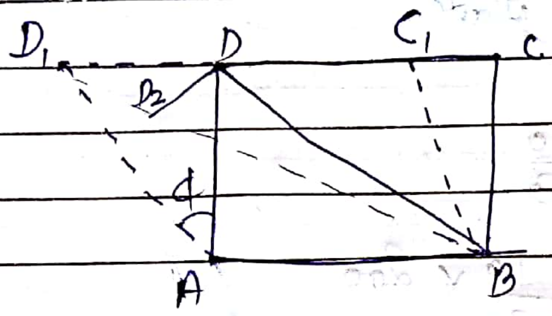
4. Q. Prove: $C = \frac{mE}{2(m+1)}$



Strain of BD $= \frac{BD_1 - BD}{BD} = \frac{D_1D_2}{BD} = \frac{DD_1 \cos 45°}{AD\sqrt{2}} = \frac{DD_1}{2AD} = \frac{\phi}{2}$

$$= \frac{\tau}{2C} \quad \text{--- (i)}$$

Tensile strain on the diagonal BD due to tensile stress on the diagonal BD $= \frac{\tau}{E}$

and the tensile strain on the diagonal BD due to compressive stress on the diagonal AC $= \frac{1}{m} \cdot \frac{\tau}{E}$

The combined effect of the above two stresses on the diagonal BD $= \frac{\tau}{E} + \frac{1}{m}\frac{\tau}{E} = \frac{\tau}{E}\left(1 + \frac{1}{m}\right)$ --- (ii)

equating equation (i) & (ii)

$$\frac{\tau}{2C} = \frac{\tau}{E}\left(\frac{m+1}{m}\right)$$

$$\Rightarrow \quad C = \frac{mE}{2(m+1)}$$

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5. A steel plate has the modulus of elasticity as 200 GPa and poisson's ratio as 0.3. What is the Value of Bulk modulus for the steel plate.

A.

$E = 200\ GPa$

$m = \frac{1}{0.3} = \frac{10}{3}$

$K = \frac{mE}{3(m-2)} = \frac{\frac{10}{3} \times 200}{3\left(\frac{10}{3} - 2\right)}$

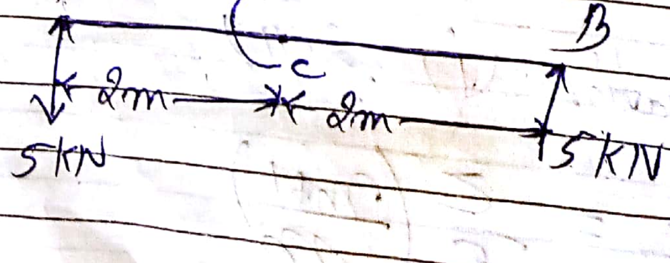
$K = \frac{10}{3} \times 200 \times \frac{3}{3 \times 4}$

$K = \frac{2000}{12}$

$K = 166.6\ GPa$ A.

6. A Simply supported Beam AB 4 m span is Subjected to a clockwise moment of 20 KN-m at its Centre. Draw the SF and BM diagram.

A.



B-C $\quad F_x = 5 \quad\quad M_x = 5 \cdot x$

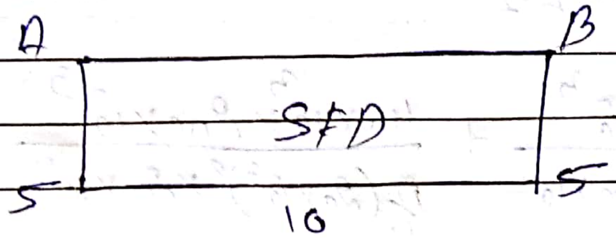
$x = 0 \quad F_B = -5\,kN \quad\quad M_B = 0$

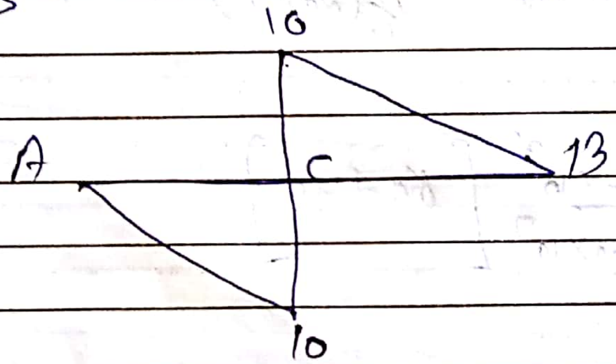
$x = 2 \quad F_C = -5\,kN \quad\quad M_C = 10\,kN\text{-}m$

C-A $\quad F_x = -5 \quad\quad M_x = 5x - 20$

$x = 2 \quad F_C = -5\,kN \quad\quad M_C = -10\,kN\text{-}m$

$x = 4 \quad F_A = -5\,kN \quad\quad M_A = 0$



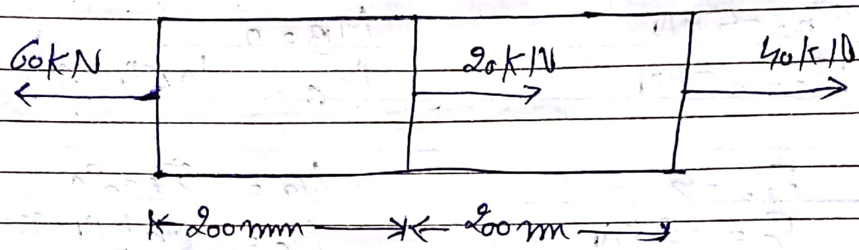


BMD

Sec-C.

1. A steel Bar ABC of 400 mm length and 20 mm diameter is subjected is subjected to a point Load as shown in figure. Determine the total change in length of the bar if $E = 200\,GPa$

A.



$$\delta l = \delta l_1 + \delta l_2$$

$$\delta l = \frac{P_1 . l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2}$$

$$\delta l = \frac{60\times10^{3}\times200\times10^{-3}}{\frac{\pi}{4}(20\times10^{-3})^2\times200\times10^{9}} + \frac{40\times10^{3}\times200\times10^{-3}}{\frac{\pi}{4}(20\times10^{-3})^2\times200\times10^{9}}$$

$$\delta l = \frac{200\times10^{-3}\times4\times10^{3}}{\pi\times400\times10^{-6}\times200\times10^{9}}\left[60+40\right]$$

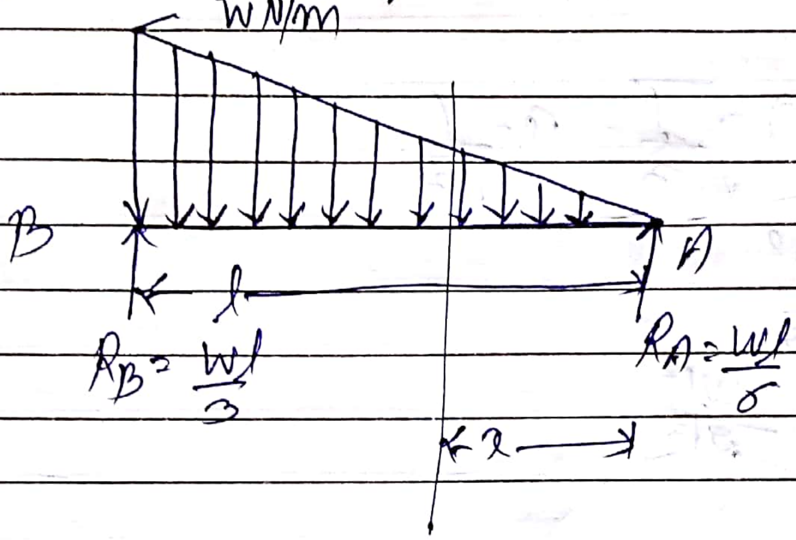
$$\delta l = \frac{8\times100}{\pi\times4\times2\times10^{5}}$$

$$\delta l = \frac{1}{\pi}\times10^{-3}$$

$$\delta l = 0.318\times10^{-3}\,m$$

$$\delta l = 0.318\ mm$$ Ans.

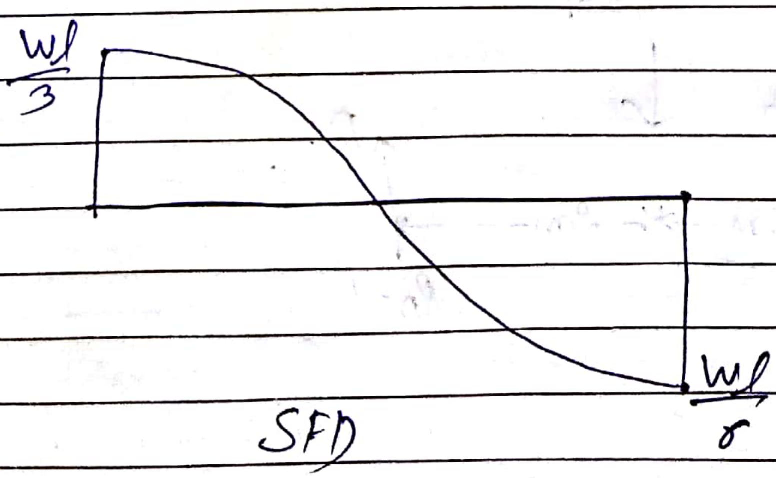
Q.2. Draw the shear force and Bending moment diagram for a Simply supported Beam loaded with a gradually Varying Load of intensity zero from one end to W N/m at the other end.



$$F_x = -\frac{Wl}{6} + \frac{1}{2} \cdot \frac{Wx}{l} \cdot x \qquad M_x = \frac{Wl}{6}x - \frac{Wx^2}{2l} \cdot \frac{x}{3}$$

$$F_A = -\frac{Wl}{6} \qquad M_A = 0$$

$$F_B = \frac{Wl}{3} \qquad M_B = 0$$



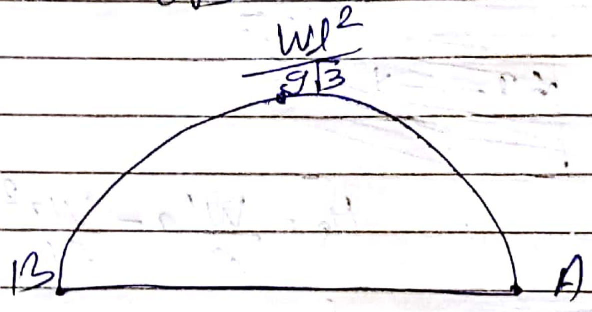
SFD

because the shear force changes sign

so that $-\frac{Wl}{6} + \frac{Wx^2}{2l} = 0$

$x = \sqrt{\frac{l^2}{3}}$, $x = \frac{l}{\sqrt{3}}$

$M_{l/\sqrt{3}} = \frac{Wl^2}{9\sqrt{3}}$



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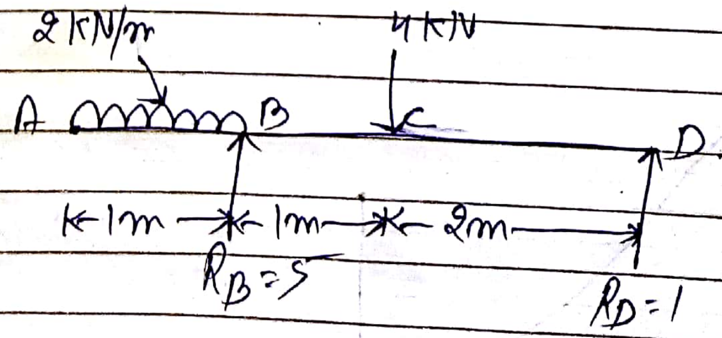
3. An overhanging beam is loaded as shown in figure. Draw the shear force and bending moment diagram and point of contraflexure, if any.

Q.



$$R_B + R_D = 6$$

$$-2 \times 1 \times \frac{1}{2} + 4 \times 1 - R_D \times 3 = 0$$

$$-1 + 4 = R_D \times 3 \Rightarrow R_D = 1$$

$$R_B = 5$$

D-C $\quad F_x = -1 \qquad M_x = 1 \cdot x$

$x = 0 \quad F_D = -1$ KN $\qquad M_D = 0$

$x = 2 \quad F_C = -1$ KN $\qquad M_C = 2$ KN-m

C-B $\quad F_x = -1 + 4 \qquad M_x = 1 \cdot x - 4(x-2)$

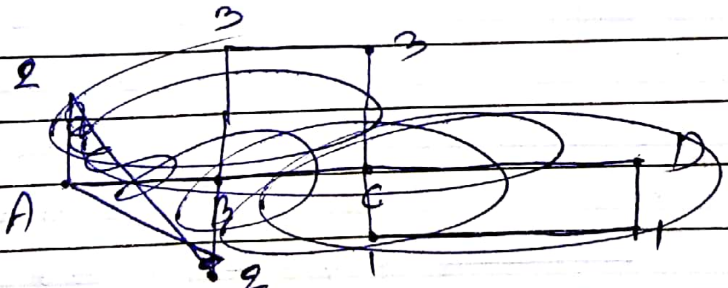
$x = 2 \quad F_C = 3$ KN $\qquad M_C = 2$ KN-m

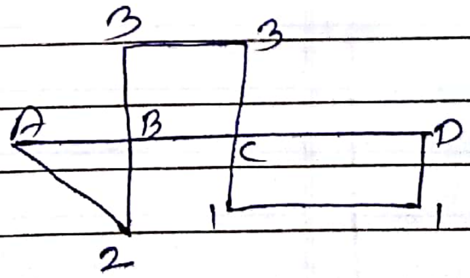
$x = 3 \quad F_B = 3$ KN $\qquad M_B = -1$ KN-m

B-A $\quad F_x = -1 + 4 - 5 + 2(x-3) \qquad M_x = 1 \cdot x - 4(x-2) + 5(x-3) - \frac{2(x-3)^2}{2}$

$x = 3 \quad F_B = -2$ KN $\qquad M_B = -1$ KN-m

$x = 4 \quad F_A = 0$ KN $\qquad M_A = 0.$

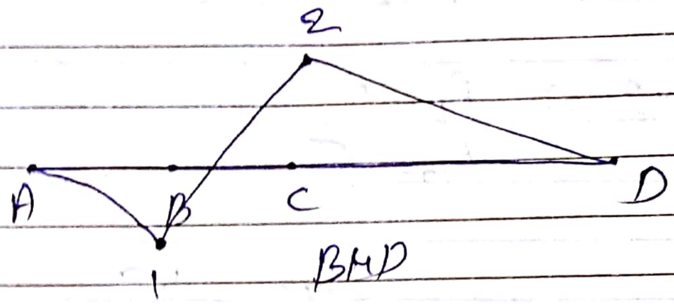




Bending Moment changes sign between B-A

$F_x = 0 \Rightarrow -1 + 4 - 5 + 2(x-3) = 0$

$-2 + 2(x-3) = 0$



BMD

The Bending Moment changes sign between C-B so that $M_{C-B}=0$

$1.x-4(x-2)=0$

$x-4x+8=0$

$-3x=-8$

$x=\frac{8}{3}=2.66$ m is the point of Contraflexure.