Rajasthan Institute of Engg & Technology

Department of Electronics & Comm.

II Year III Sem.

Signal & Systems

(Solutions of I Mid-Term)

Set 'A'

Sec. A

1. $x(t) = t\,u(t)$

$\because$ Energy $E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt$

$$= \lim_{T\to\infty} \int_0^{T/2} t^2 dt = \lim_{T\to\infty} \frac{(T/2)^3}{3} = \infty \quad \text{--- (1)}$$

Power $P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^{T/2} t^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \cdot \frac{T^3}{24} = \infty. \quad \text{--- (2)}$$

From (1) & (2), given signal is neither an energy nor a power signal.

2. Time-shifting property of DTFT:-

$\because \quad x(n) \longleftrightarrow X(e^{jw})$

then $x(n-n_0) \longleftrightarrow e^{-jwn_0} X(e^{jw})$.

Proof: $\quad \text{DTFT } x(n) = X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jwn}$

$\therefore \text{DTFT of } x(n-n_0) = \sum_{n=-\infty}^{\infty} x(n-n_0) e^{-jwn}$

Put $n-n_0 = m$, so $n = m+n_0$

$$\text{DTFT } x(n-n_0) = \sum_{m=-\infty}^{\infty} x(m) e^{-jw(m+n_0)}$$

$$= \sum_{m=-\infty}^{\infty} x(m) e^{-jwm} \cdot e^{-wn_0}$$

$$= X(e^{jw}) \cdot e^{-jwn_0}.$$

$$\therefore \quad x(n-n_0) \longleftrightarrow e^{-jwn_0} X(e^{jw})$$

3. Inverse Fourier transform of $\delta(w-w_0)$

$\because \quad F^{-1}[X(w)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w) e^{jwt} dw$

or $\quad F^{-1}[\delta(w-w_0)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(w-w_0) e^{jwt} dw.$

Using shifting property,

$$F^{-1}[\delta(w-w_0)] = \frac{1}{2\pi}\left[e^{jwt}\right]_{w=w_0} = \frac{1}{2\pi} e^{jw_0 t}$$

$$F\left[\frac{1}{2\pi} e^{j\omega_0 t}\right] = \delta(\omega - \omega_0).$$

or $\frac{1}{2\pi} e^{j\omega_0 t} \longleftrightarrow \delta(\omega - \omega_0).$

or $e^{j\omega_0 t} \longleftrightarrow 2\pi\,\delta(\omega - \omega_0).$

(4.) Relationship between Unit Step, Ramp & Impulse/ Delta function.

Unit step → $u(t) = \begin{cases} 1 & t > 0 \\ 0 & \text{else} \end{cases}$

Ramp → $r(t) = \begin{cases} t & t > 0 \\ 0 & \text{else} \end{cases}$

Impulse → $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{else} \end{cases}$

∴ (1) $\frac{d}{dt} r(t) = u(t)$ or

or $\int u(t)\,dt = r(t)$

(2) $\frac{d}{dt} u(t) = \delta(t)$

or $\int \delta(t)\,dt = u(t)$

(3.) $r(t) = \iint \delta(t)\,dt.$ or

$\frac{d^2}{dt^2} r(t) = \delta(t)$

$\therefore$ $\delta(t) \xleftarrow{\text{Integrate}} u(t) \xleftarrow{\text{Integrate}} r(t)$

$r(t) \xleftarrow{\text{Differentiate}} u(t) \xleftrightarrow{\text{Differentiate}} \delta(t)$

⑤ Properties of unit - Impulse function

1. Shifting Property.

$$\int_{-\infty}^{\infty} x(t)\,\delta(t - t_m)\,dt = x(t_m)$$

2. Replication Property

$$x(t) \otimes \delta(t) = x(t).$$

Sec - B

1. $y(t) = x^2(t - t_0) + 2$

(a). For linearity —

$y_1(t) = x^2(t - t_0) + 2$

$y_2(t) = x^2(t - t_0) + 2$

A linear Combination of two I/p results in

$y_3(t) = T[x_3^2(t - t_0) + 2]$

$= T[\{Ax_1(t - t_0) + Bx_2(t - t_0)\}^2 + 2]$

$= A^2x_1^2(t - t_0) + B^2x_2^2(t - t_0) + 2ABx_1(t - t_0)x_2(t - t_0) + 2$ —— ①

A linear combination of two o/p results in

$$A y_1(t) + B y_2(t).$$

so $A y_1(t) + B y_2(t) = A\left[x_1^2(t-t_0)+2\right] + B\left[x_2^2(t-t_0)+2\right]$

$$A y_1(t) + B y_2(t) = A x_1^2(t-t_0) + B x_2^2(t-t_0) + 2(A+B) \quad (2)$$

$\therefore$ (1) $\neq$ (2),

So System is Non-linear.

<u>Causal :-</u> $y(t) = x^2(t-t_0) + 2$

Causal, since output does not depend upon future value of I/p signal; but depends on past value of I/p.

<u>Stable :-</u> $y(t) = x^2(t-t_0) + 2$

at $x(t) = 1$,

$y(t) = 1+2 = 3.$

Bounded I/p :- Bounded O/p. so stable.

2. Convolution of $x(t) = 3\cos 2t$ &

$h(t) = e^{-|t|}$

$$y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(\tau) = 3\cos 2\tau$ for all τ.

$h(t-\tau) = e^{(t-\tau)}$ for $t-\tau < 0$ or $\tau > t$

$= e^{(\tau-t)}$ for $t-\tau \geq 0$ or $\tau \leq t$

=

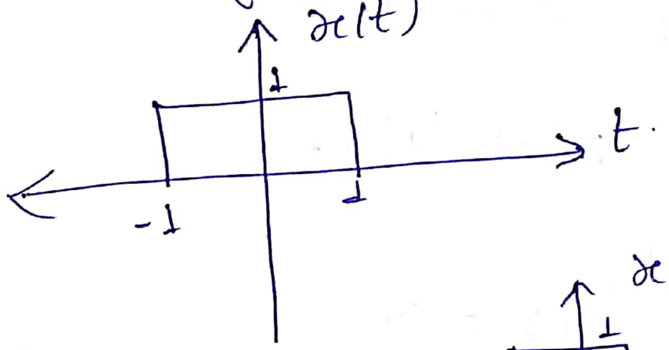
$$y(t) = \int_{\tau=-\infty}^{t} 3\cos 2\tau \, e^{(\tau-t)} d\tau + \int_{\tau=t}^{\infty} 3\cos 2\tau \, e^{(t-\tau)} d\tau$$

$$= 3e^{-t} \int_{\tau=-\infty}^{t} 3\cos 2\tau \cdot e^{\tau} d\tau + 3e^{t} \int_{\tau=t}^{\infty} e^{-\tau} \cos 2\tau \, d\tau$$

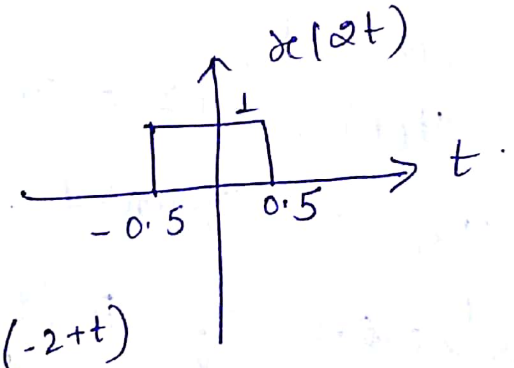
$$= 3e^{-t} \left[\frac{e^{\tau} \cos 2\tau + 2\sin 2\tau}{5} \right]_{-\infty}^{t} + 3e^{t} \left[e^{-\tau} (-\cos 2\tau + 2\sin 2\tau) \right]_{t}^{\infty}$$

$$= \frac{6}{5} \cos 2t$$

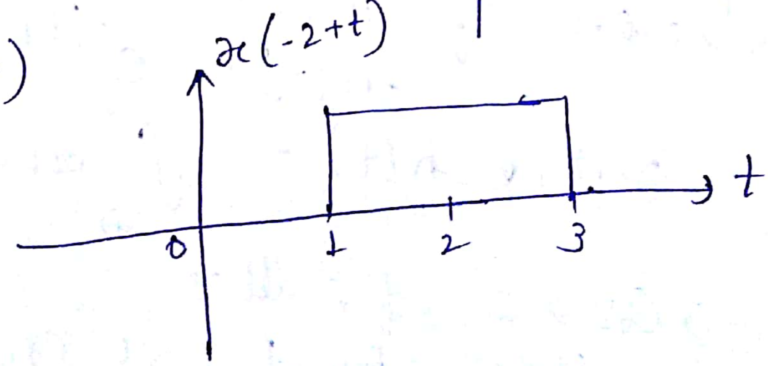
(3.) $x(t) = \begin{cases} 1 & \text{for } -1 \leq t \leq 1 \\ 0 & \text{else.} \end{cases}$



(a.) $x(2t) \longrightarrow$



(b) $x(-2+t)$



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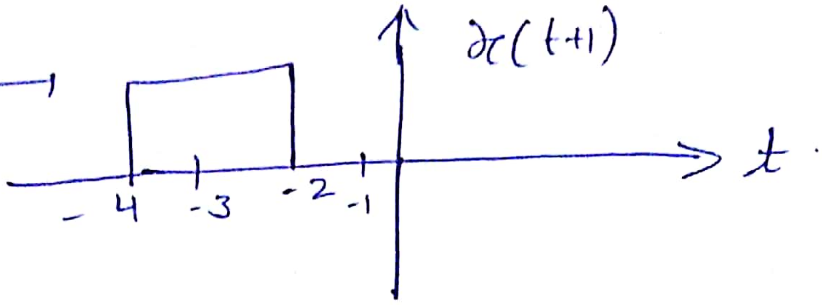
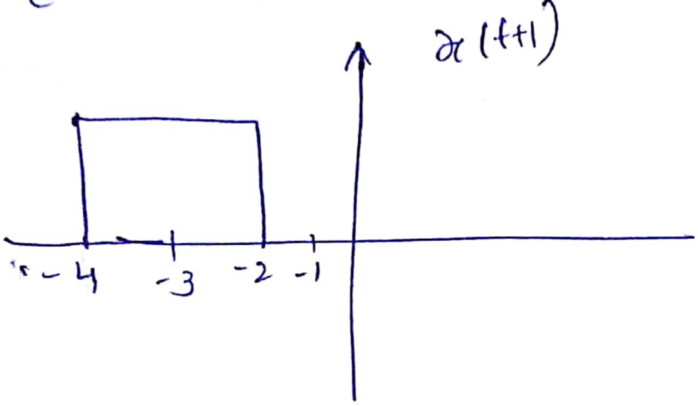
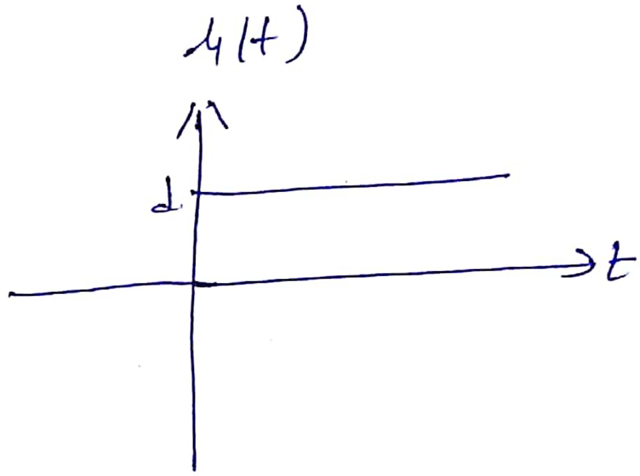
(c) $x(t+1) \rightarrow$



(d.) $x(t+1)\,u(t)$





$\Downarrow$

$$y(t) = 0.$$

4.

5. Properties of CTFT.

(a) Scaling :-

If $x(t) \longleftrightarrow X(\omega)$

then $x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

Proof :-

$\because X(\omega) = F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$

$\therefore \quad F[x(at)] = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$

Put $at = y \Rightarrow dt = dy/a$

$\therefore F[x(at)] = \int_{-\infty}^{\infty} x(y)\, e^{-j\left(\frac{\omega}{a}\right)\cdot y}\, \frac{dy}{a}$

$= \frac{1}{a} \int_{-\infty}^{\infty} x(y)\, e^{-j\left(\frac{\omega}{a}\right) y}\, dy$

$= \frac{1}{a} X\left(\frac{\omega}{a}\right)$

& For (-)ve a,

$F[x(at)] = \frac{-1}{a} X\left(\frac{\omega}{a}\right)$

$\therefore \quad F[x(at)] = \frac{1}{|a|} X\left(\frac{\omega}{a}\right).$

(b.) Time-shifting :-

If $x(t) \longleftrightarrow X(\omega).$

then $x(t-b) = X(\omega)\, e^{-j\omega b}$

Proof:-

$$\because X(\omega) = F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$\therefore F[x(t-b)] = \int_{-\infty}^{\infty} x(t-b)\, e^{-j\omega t} dt.$$

Put $t - b = y \Rightarrow dt = dy$

$$F[x(t-b)] = \int_{-\infty}^{\infty} x(y)\, e^{-j\omega(b+y)} \cdot dy$$

$$= e^{-j\omega b} \int_{-\infty}^{\infty} x(y)\, e^{-j\omega y} dy$$

$$F[x(t-b)] = e^{-j\omega b} X(\omega).$$

$$\therefore x(t-b) \longleftrightarrow e^{-j\omega b} X(\omega)$$

(c) Time Convolution:-

If $x_1(t) \longleftrightarrow X_1(\omega)$ & $x_2(t) \longleftrightarrow X_2(\omega)$

then $x_1(t) * x_2(t) \longleftrightarrow X_1(\omega) \cdot X_2(\omega)$

Proof:-

$$F[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$\therefore F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(t) * x_2(t)\, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau) \right] e^{-j\omega t} \, dt \, d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \int_{-\infty}^{\infty} x_2(t-\tau)\, e^{-j\omega t} \, dt \, d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, e^{-j\omega \tau} X_2(\omega)\, d\tau$$

$$= X_1(\omega) \cdot X_2(\omega).$$

$$\therefore x_1(t) * x_2(t) \longleftrightarrow X_1(\omega) \cdot X_2(\omega)$$

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(d.) Integration :-

If $x(t) \longleftrightarrow X(\omega)$ then

$$\int_{-\infty}^{t} x(\tau)\, d\tau = \frac{1}{j\omega} X(\omega)$$

Proof :- ~~$F[x(t)] =$~~

$$\because x(t) = \frac{d}{dt}\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right]$$

$$\therefore F[x(t)] = F\left[\frac{d}{dt}\left\{\int_{-\infty}^{t} x(\tau) d\tau\right\}\right]$$

Using differentiation property,

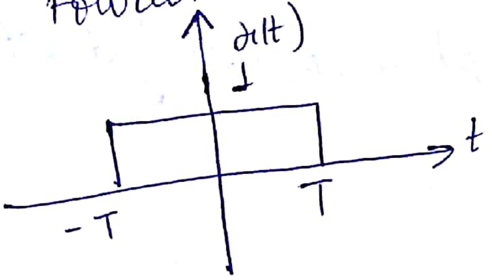
$$F[x(t)] = j\omega \left[F\left\{\int_{-\infty}^{t} x(\tau)\, d\tau\right]$$

$$X(\omega) = F[x(t)] = j\omega\, F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right]$$

$$\frac{1}{j\omega} X(\omega) = F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right]$$

$$\therefore F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right] = \frac{1}{j\omega} X(\omega).$$

6. Fourier transform of Rectangular Pulse :-



$$\because X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

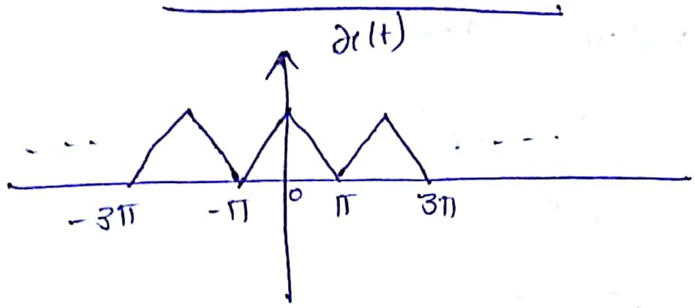
$$= \int_{-T}^{T} 1 \cdot e^{-j\omega t} dt$$

$$X(\omega) = \frac{1}{-j\omega}\left[e^{-j\omega T} - e^{j\omega T}\right]$$

$$X(\omega) = \frac{2}{\omega}\left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right] = \frac{2}{\omega} \sin(\omega T)$$

Sec 'C'

Q1.



$$x(t) = \frac{A}{\pi}(\omega t + \pi) \quad \text{for } -\pi \le \omega t \le 0$$

$$= -\frac{A}{\pi}(\omega t - \pi) \quad \text{for } 0 \le \omega t \le \pi$$

Let $t_0 = -\pi$, then $t_0 + T = \pi$

or $w_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 1.$

$\therefore x(\omega t) = x(-\omega t)$ so $b_n = 0.$

$$a_0 = \frac{2}{T}\int_0^{T/2} x(t)\, d(\omega t) = \frac{2}{2\pi}\int_{-\pi}^{0} \frac{A}{\pi}(\omega t + \pi)\, d(\omega t).$$

$$a_0 = A/2$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t) \cos n\omega t \, d(\omega t).$$

$$= \frac{4}{2\pi}\int_{-\pi}^{0} \frac{A}{\pi}(\omega t + \pi) \cos n\omega t \, d(\omega t)$$

$$= \frac{2A}{\pi}\int_{-\pi}^{0} \left\{ \frac{1}{\pi}\omega t \cos n\omega t + \cos n\omega t \right\} d(\omega t)$$

$$= \frac{2A}{n^2\pi^2}\left[1 - (-1)^n\right]$$

$$\therefore a_n = \begin{cases} \frac{4A}{n^2\pi^2} & \text{for odd } n \\ 0 & \text{for even } n. \end{cases}$$

(d.) Integration :-

If $x(t) \longleftrightarrow X(\omega)$ then

$$\int_{-\infty}^{t} x(\tau)\, d\tau = \frac{1}{j\omega} X(\omega)$$

Proof :- ~~$\therefore F[x(t)] =$~~

$$\because\ x(t) = \frac{d}{dt}\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right]$$

$$\therefore F[x(t)] = F\left[\frac{d}{dt}\left\{\int_{-\infty}^{t} x(\tau)\, d\tau\right\}\right]$$

Using differentiation property,

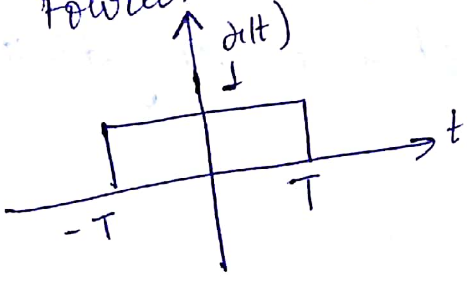
$$F[x(t)] = j\omega \left[F\left\{\int_{-\infty}^{t} x(\tau)\, d\tau\right]\right.$$

$$X(\omega) = F[x(t)] = j\omega\, F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right]$$

$$\frac{1}{j\omega} X(\omega) = F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right]$$

$$\therefore\ F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right] = \frac{1}{j\omega} X(\omega).$$

6. Fourier transform of Rectangular Pulse :-



$$\because\ X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T}^{T} 1 . e^{-j\omega t}\, dt$$

$$X(\omega) = \frac{1}{j\omega}\left[e^{-j\omega T} - e^{j\omega T}\right]$$

$$X(\omega) = \frac{2}{\omega}\left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right] = \frac{2}{\omega} \sin(\omega T)$$

(2) $y(n) - \frac{3y(n-1)}{4} + \frac{y(n-2)}{8} = 2x(n)$

DTFT on both sides, we get

$$Y(e^{jw}) - \frac{3}{4} e^{-jw} Y(e^{jw}) + \frac{1}{8} e^{-2jw} Y(e^{jw}) = 2X(e^{jw})$$

$$Y(e^{jw}) \left[1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-2jw} \right] = 2X(e^{jw})$$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{2}{1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-2jw}} = \text{Freq. Response}$$

To get Impulse response, IDFT of $H(e^{jw})$.

$$h(n) = \text{IDFT} \left[\frac{2}{1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-2jw}} \right]$$

$$= \text{IDFT} \left[\frac{2}{\left(1 - \frac{1}{2} e^{-jw}\right)\left(1 - \frac{1}{4} e^{-jw}\right)} \right] = \text{IDFT} \left[\frac{A}{1 - \frac{1}{2} e^{-jw}} + \frac{B}{1 - \frac{1}{4} e^{-jw}} \right]$$

$$= \text{IDFT} \left[\frac{4}{1 - \frac{1}{2} e^{-jw}} - \frac{2}{1 - \frac{1}{4} e^{-jw}} \right]$$

$$h(n) = 4 \cdot \left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{4}\right)^n u(n). \quad \text{Ans}$$

(3) $x_1(n) = \left(\frac{1}{2}\right)^n u(n)$

$x_2(n) = (1/3)^n u(n)$

Convolution + $x_1(n) \otimes x_2(n) = X_1(w) \cdot X_2(w)$

or ~~R(n)~~ $x_3(n) = \text{IDFT}[X_1(w) X_2(w)]$

$$X_1(w) = \frac{1}{1-\frac{1}{2}e^{-jw}}$$

$$X_2(w) = \frac{1}{1-\frac{1}{3}e^{-jw}}$$

$$\therefore \quad x_1(n) * x_2(n) \longleftrightarrow X_1(w) \cdot X_2(w)$$

$$= \frac{1}{1-\frac{1}{2}e^{-jw}} \cdot \frac{1}{1-\frac{1}{3}e^{-jw}}$$

$$= \frac{A}{1-\frac{1}{2}e^{-jw}} + \frac{B}{1-\frac{1}{3}e^{-jw}}$$

$\cancel{A = 1/3}$, $\quad B = -2$

$A = 3$,

$$\therefore \quad = \frac{3}{1-\frac{1}{2}e^{-jw}} - \frac{2}{1-\frac{1}{3}e^{-jw}}$$

$$\cancel{h(n) =} \quad x_3(n) = \text{IDFT}\left[\frac{3}{1-\frac{1}{2}e^{-jw}} - \frac{2}{1-\frac{1}{3}e^{-jw}}\right]$$

$$x_3(n) = 3 \cdot \left(\frac{1}{2}\right)^n u(n) - 2 \cdot \left(\frac{1}{3}\right)^n u(n)$$

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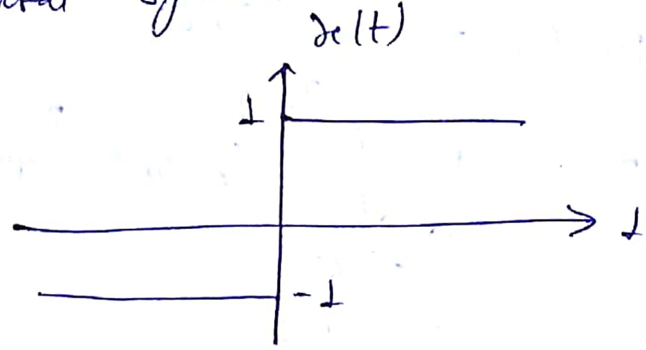
Rajasthan Institute of Engg & Technology
Department of Electronics & Commn
II Year III Sem
Signal & Systems
I Mid Term Solutions.

Set 'B'

Sec 'A'

1. $y(n) = 3x(n-2) + 3x(n+2)$
Non- Causal system.

2. Signum function



$$x(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$

3. Time - Differentiation Property of CTFT :-

$$x(t) \longleftrightarrow X(\omega)$$
$$\frac{dx(t)}{dt} \longleftrightarrow j\omega X(\omega)$$

Proof :-

$$F^{-1}[X(\omega)] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\left[\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega\right]$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{d}{dt} X(\omega) e^{j\omega t} \cdot d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega X(\omega) e^{j\omega t} d\omega = F^{-1}[j\omega X(\omega)]$$

$$\therefore F\frac{d(x(t))}{dt} = j\omega X(\omega).$$

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(4.) Fourier transform of $\cos w_0 t$.

$$\therefore \cos w_0 t = \frac{e^{jw_0 t} + e^{-jw_0 t}}{2}$$

$$\therefore e^{jw_0 t} \longleftrightarrow 2\pi\delta(w - w_0)$$

$$e^{-jw_0 t} \longleftrightarrow 2\pi\delta(w + w_0).$$

$$\therefore \cos w_0 t \longleftrightarrow \frac{1}{2}\left[2\pi\delta(w - w_0) + 2\pi\delta(w + w_0)\right]$$

$$\cos w_0 t \longleftrightarrow \left[\pi\delta(w - w_0) + \pi\delta(w + w_0)\right].$$

(5.) $x(t) = A \sin t$

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A\sin t)^2 \, dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2 \sin^2 t \, dt.$$

$$= \frac{A^2}{T_0}\int_{-T_0/2}^{T_0/2} \frac{1 - \cos 2t}{2} \, dt.$$

$$= \frac{A^2}{2T_0}\left[t - \frac{\sin 2t}{2}\right]_{-T_0/2}^{T_0/2} = \frac{A^2}{2}.$$

$$E = \int_{-\infty}^{\infty} (A\sin t)^2 \, dt = \int_{-\infty}^{\infty} A^2 \sin^2 t \, dt \Rightarrow E = \infty$$

so It is a Power Signal

$$P = A^2/2.$$

Sec. - 'B'

3

Q1.

Properties of DTFT :-

(a.) Scaling :- If $x(n) \longleftrightarrow X(e^{jw})$

then $y(n) = x(P.n) \longleftrightarrow X\left(e^{\frac{jw}{P}}\right)$.

(b) Time - Shifting :-

If $x(n) \longleftrightarrow X(e^{jw})$

then $x(n-n_0) \longleftrightarrow e^{-jwn_0} X(e^{jw})$

(c) Time Convolution :-

If $x(n) \longleftrightarrow X(e^{jw})$ &

$y(n) \longleftrightarrow Y(e^{jw})$ then

$z(n) = x(n) \otimes y(n) \longleftrightarrow Z(e^{jw}) = X(e^{jw}).Y(e^{jw})$

(d) Freq. Shifting :-

If $x(n) \longleftrightarrow X(e^{jw})$

then $e^{jw_0 n} \longleftrightarrow X\left[e^{j(w-w_0)}\right]$

2. $y(t) = \sin[x(t+2)]$

∴ Not Memoryless, static.

Non Causal.

Time-Invariant, Non-linear, Stable.

(3) $e^{-3t}u(t)$, & $h(t) = u(t-1)$

Convolution.

$\because\ y(t) = x(t) * h(t)$

$$= \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(\tau) = e^{-3\tau}u(\tau)$ & $h(t-\tau) = u(t-\tau-1)$.

$$y(t) = \int_{-\infty}^{\infty} e^{-3\tau}u(\tau) \cdot u(t-\tau-1)\, d\tau$$

$u(\tau) = 1$; $\tau \geq 0$.

$u(t-\tau-1) = 1$; $t-1 \geq \tau$ or. $\tau \leq t-1$

$$\therefore\ y(t) = \int_{0}^{t-1} e^{-3\tau} \cdot 1 \cdot 1\, d\tau$$

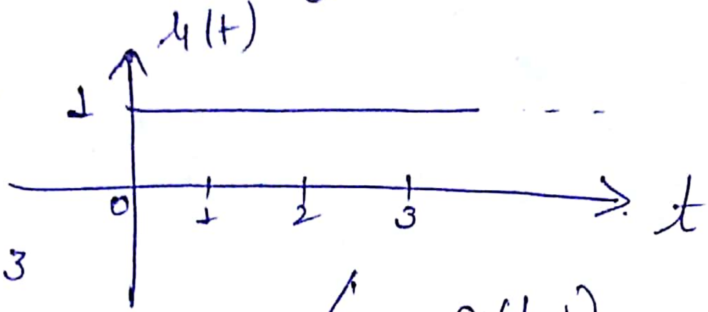
$$= \left[\frac{e^{-3\tau}}{-3}\right]_0^{t-1} = \frac{-1}{3}\left[e^{-3(t-1)} - e^{-0}\right]$$

$$= \frac{-1}{3}\left[e^{-3(t-1)} - 1\right]$$

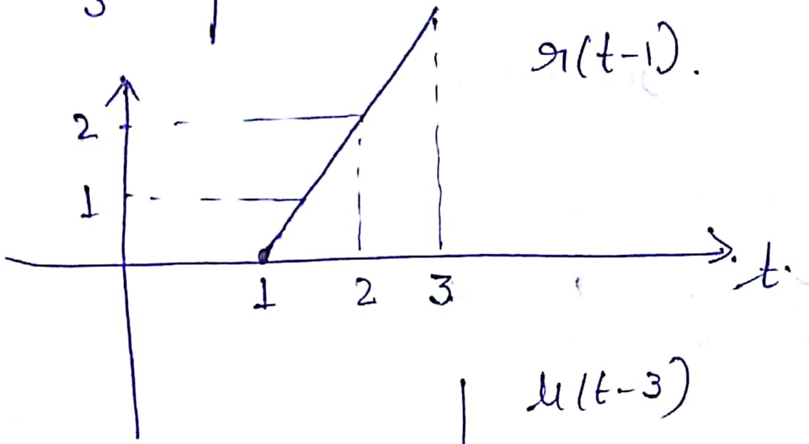
$$y(t) = \frac{1}{3}\left[1 - e^{-3(t-1)}\right]$$

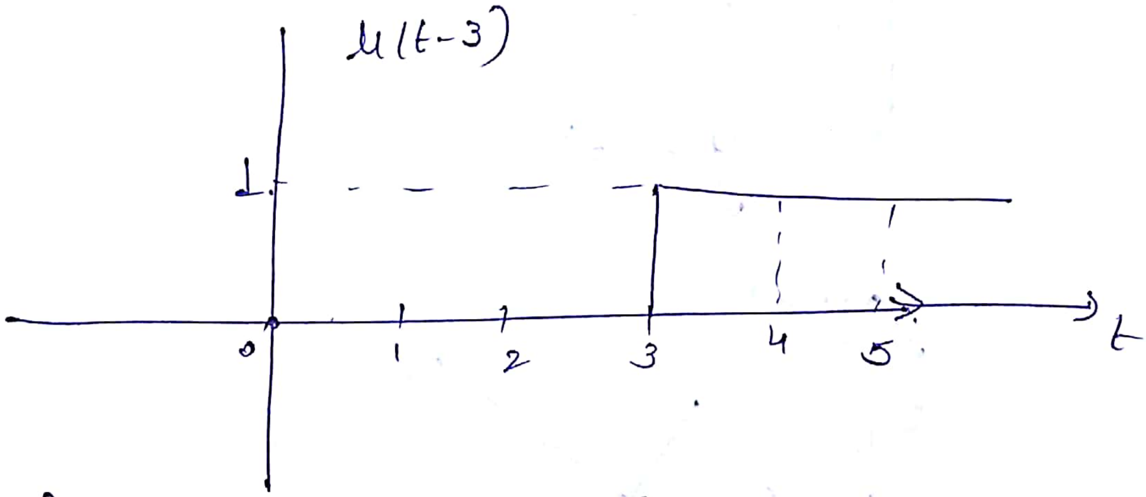
(4.) $x(t) = [u(t) + r(t-1) - 2u(t-3)] \cdot u(-t+5)$ (5)

Sol^n

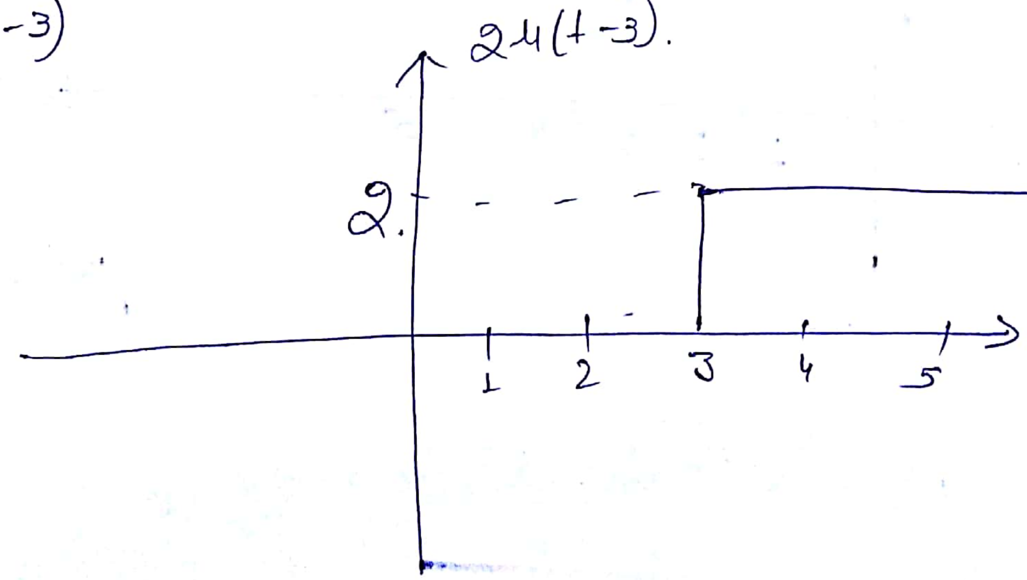


3

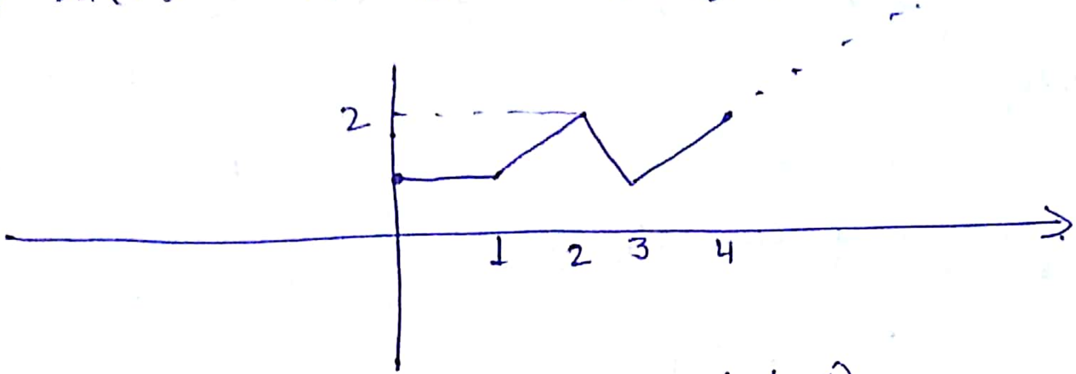




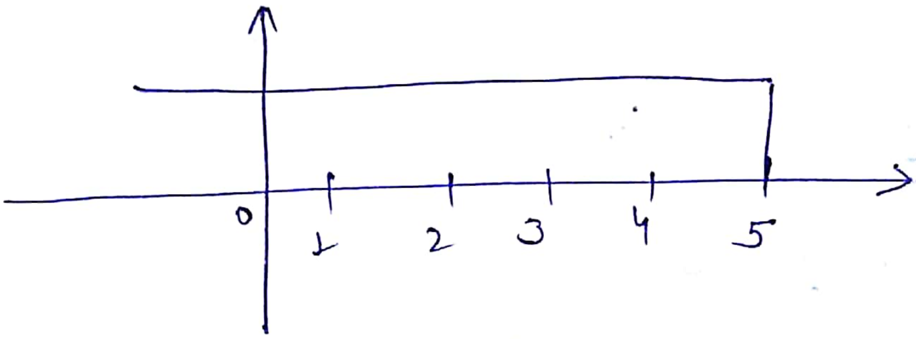
$2u(t-3)$



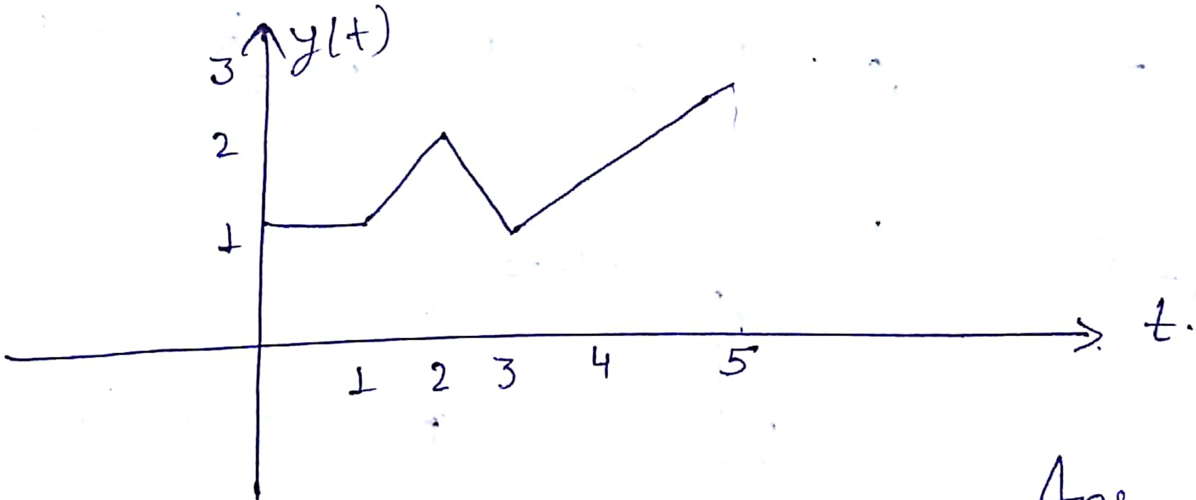
so $u(t) + r(t-1) - 2u(t-3)$



$u(-t+5)$.



$y(t) \Rightarrow$



Ans.

(5) $\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$. (7)

If $x(t) = t\,e^{-2t}u(t)$. — $y(t) = ?$

Soln $(jw)^2 Y(w) + 6jw\,Y(w) + 8Y(w) = 2X(w)$

$Y(w)\left[(jw)^2 + 6jw + 8\right] = 2X(w)$

$$H(w) = \frac{Y(w)}{X(w)} = \frac{2}{(jw)^2 + 6jw + 8}$$

$$= \frac{2}{((jw)^2 + 6jw + 8)}.$$

Now $X(w) = \frac{1}{(2+jw)^2}$

so $Y(w) = \frac{2}{((jw)^2 + 6jw + 8)} \cdot \frac{1}{(2+jw)^2}$

$$Y(w) = \frac{2}{(2+jw)(4+jw)(2+jw)^2}$$

$$= \frac{A}{jw+4} + \frac{B}{jw+2} + \frac{C}{(jw+2)^2} + \frac{D}{(jw+2)^3}$$

$$Y(w) = \frac{1/32}{jw+4} + \frac{1}{jw+2} + \frac{1}{(jw+2)^2} + \frac{1}{(jw+2)^3}$$

so $y(t) = \frac{1}{32}e^{-4t}u(t) + e^{-2t}u(t) + t\cdot e^{-2t}u(t)$
$+ \frac{t^2}{2}\cdot e^{-2t}u(t)$ Ans.

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(6.) Fourier transform of Gate function:-

$$\text{rect}\left(\frac{t}{T}\right) = A \quad \text{for } -T/2 \le t \le T/2$$
$$0 \quad \text{else.}$$

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-T/2}^{T/2} A \cdot e^{-j2\pi f t}\, dt$$

$$= \frac{-A}{j2\pi f}\left[e^{-j\pi f T} - e^{+j\pi f T}\right]$$

$$= \frac{A}{\pi f}\left[\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j}\right]$$

$$= \frac{A}{\pi f} \sin(\pi f T)$$

$$= AT \cdot \frac{\sin \pi f T}{\pi f T}$$

$$F[\text{rec}(t/T)] = AT\, \text{Sinc}(fT) \quad \text{Ans.}$$

Sec 'c'

2. $$y(n) - \frac{3y(n-1)}{5} + \frac{y(n-2)}{4} = 2x(n).$$

Freq Response –

$$Y(e^{jw}) - \frac{3}{5} e^{-jw} Y(e^{jw}) + \frac{1}{4} e^{-2jw} Y(e^{jw}) = X(e^{jw}) \cdot 2$$

$$\frac{Y(e^{jw})}{X(e^{jw})} = H(e^{jw}) = \frac{2}{1 - \frac{3}{5} e^{-jw} + \frac{1}{4} e^{-2jw}}.$$

$$h(n) = F^{-1}\left[H(e^{jw})\right]$$

$$= F^{-1}\left[\frac{2}{1 - \frac{3}{5} e^{-jw} + \frac{1}{4} e^{-2jw}}\right]$$

$$h(n). = F^{-1}\left[\frac{2}{1 - \frac{3}{5} e^{-jw} + \frac{1}{4} e^{-2jw}}\right]$$

(3.) $x_1(n) = \left(\frac{1}{4}\right)^n u(n)$

$x_2(n) = \left(\frac{1}{5}\right)^n u(n)$

$$X_1(w) = \frac{1}{1 - \frac{1}{4} e^{-jw}}$$

$$X_2(w) = \frac{1}{1 - \frac{1}{5} e^{-jw}}$$

$$X(w) = x_1(n) * x_2(n) = X_1(w) \cdot X_2(w)$$

$$= \frac{1}{\left(1 - \frac{1}{4} e^{-jw}\right)\left(1 - \frac{1}{5} e^{-jw}\right)}$$

$$= \frac{A_1}{1 - \frac{1}{4} e^{-jw}} + \frac{A_2}{1 - \frac{1}{5} e^{-jw}}$$

$A_1 = 5, \quad A_2 = -4.$

$$X(w) = \frac{5}{1 - \frac{1}{4} e^{-jw}} - \frac{4}{1 - \frac{1}{5} e^{-jw}}$$

$$x(n) = 5 \cdot \left(\frac{1}{4}\right)^n u(n) - 4 \cdot \left(\frac{1}{5}\right)^n u(n)$$