Rajasthan Institute of Engineering & Technology, Jaipur

University Roll No. _____

1st year MBA. Semester II I Mid Term Examination, March– 2018

Subject: -Operations Research SET-A

Time: - 2 Hrs. [Maximum Marks: -20]

[Min. Passing Marks: 08]

Instructions to the Candidates:

Attempt any 4 questions from Section A and Section B is Compulsory.

Section A

1. A firm manufacturing two types of electric items A and B, can make a profit of Rs. 20 per unit of A and Rs. 30 per unit of B. Each unit of A requires 3 motors and 2 transformers and each unit of B requires 2 motors and 4 transformers. The total supply of these per month is restricted to 210 motors and 300 transformers. Type B is an export model requiring a voltage stabilizer which has a supply restiricted to 65 units per month. Formulate the linear programming problem for maximum profit.
2. Construct the dual of the problem

$$Max.\ z = 2x_1 + 3x_2 + x_3$$

subject to

$$4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

and $x_1, x_2, x_3 \geq 0$

3. There are five jobs to be assigned, one each to five machines and the associated cost matrix is as follows. Solve the following assignment problem:

		Machines				
		I	II	III	IV	V
Jobs	A	11	17	8	16	20
	B	9	7	12	6	15
	C	13	16	15	12	16
	D	21	24	17	28	26
	E	14	10	12	11	15

4.find the initial basic feasible solution to the following transportation problem using North West corner rule.

	D_1	D_2	D_3	D_4	Available units
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Demand	20	40	30	10	100

5. Solve the following LPP by Graphical method

$$Max.Z = 5x_1 + 3x_2$$
$$S.T.\ \ 3x_1 + 5x_2 \le 15$$
$$5x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0$$

6. write the difference between Transportation and Assignment methods.

Section B

1.solve the following transportation problem for total minimum transportation cost using

Vogel's approximation method and Modi method

	D_1	D_2	D_3	Available units
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

2. A physician purchases a particular medicine on Monday of each week. The medicine must be used within the week following, otherwise it becomes worthless. The medicine costs Rs. 2 per dose and the physician charges Rs. 4 per dose. In the past 50 weeks, the records of uses are as follows:

Dose per week :	20	25	40	60
No. of wees :	5	15	25	5

Calculate (i) Expected Monetary Value (EMV)(ii) Expected Opportunity Loss (EOL) (iii) Expected Value Of Perfect Information (EVPI).

Solution: operations Research

MBA II^nd sem.

Set - A

Section A

Q.① Sol.

Requirement	Electric items A	Electric items B	Total availability
Motors	3	2	210
Transformers	2	4	300
Voltage stabilizer		1	65
profit	20	30	

Let x_1, x_2 unit produced of electric items A and B respectively.

Objective function

$$\text{Max } Z = 20x_1 + 30x_2$$

Subject to Constraint

$$3x_1 + 2x_2 \leq 210$$

$$2x_1 + 4x_2 \leq 300$$

$$x_2 \leq 65$$

$$x_1, x_2 \geq 0.$$

Q. (2) Sol. The given problem can be written as

$$\text{Max } Z_p = 2x_1 + 3x_2 + x_3$$

s.t.

$$4x_1 + 3x_2 + x_3 \le 6$$
$$-4x_1 - 3x_2 - x_3 \le -6$$
$$x_1 + 2x_2 + 5x_3 \le 4$$
$$-x_1 - 2x_2 - 5x_3 \le -4$$
$$x_1, x_2, x_3 \ge 0.$$

Then its dual is given by

$$\text{Min } Z_D = 6w_1 - 6w_2 + 4w_3 - 4w_4$$

s.t.

$$4w_1 - 4w_2 + w_3 - w_4 \ge 2$$
$$3w_1 - 3w_2 + 2w_3 - 2w_4 \ge 3$$
$$w_1 - w_2 + 5w_3 - 5w_4 \ge 1$$
$$w_1, w_2, w_3 \ge 0.$$

Let $w_1 - w_2 = w_1'$, $w_3 - w_4 = w_2'$

Then final dual

$$\text{Min } Z_D = 6w_1' + w_2'$$

s.t. $4w_1' + w_2' \ge 2$
$3w_1' + 2w_2' \ge 3$
$w_1' + 5w_2' \ge 1$ and $w_1', w_2' \ge 0.$

(3)

Q. ③ Sol:

	I	II	III	IV	V
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

Step I: Row Reduction

	I	II	III	IV	V
A	3	9	0	8	12
B	3	1	6	0	9
C	1	4	3	0	4
D	4	7	0	11	9
E	4	0	2	1	5

Step II: Column Reduction

	I	II	III	IV	V	
A	2	9	[0]	8	8	✓ ③
B	~~2~~	1	~~6~~	[0]	5	
C	[0]	4	~~3~~	~~✗~~	~~✗~~	
D	3	7	~~✗~~	11	5	✓ ①
E	~~3~~	[0]	2	~~1~~	~~1~~	
			✓ ②			

(4)

	I	II	III	IV	V
A	[11]	7	~~8~~	6	6
B	2	1	8	[0]	5
C	~~8~~	4	5	~~8~~	[0]
D	1	5	[0]	9	3
E	3	[0]	4	1	1

Optimal assignment:

	Cost
A ⟶ I	= 11
B ⟶ IV	= 6
C ⟶ V	= 16
D ⟶ III	= 17
E ⟶ II	= 10
Total Cost	= 60 Rs.

Q. (4) Sol:

	D_1	D_2	D_3	D_4	Available units
O_1	[20] 11	[10] 2	1	4	30/10/0
O_2	3	[20] 3	[20] 2	1	50/20/0
O_3	4	2	[10] 5	[10] 9	20/10/0
Demand	20/0	40/20/0	30/10/0	10/0	100

Total cost :- $20+20+90+40+50+90$

$= 310$ Rs.

Q.(5) Sol: Objective function

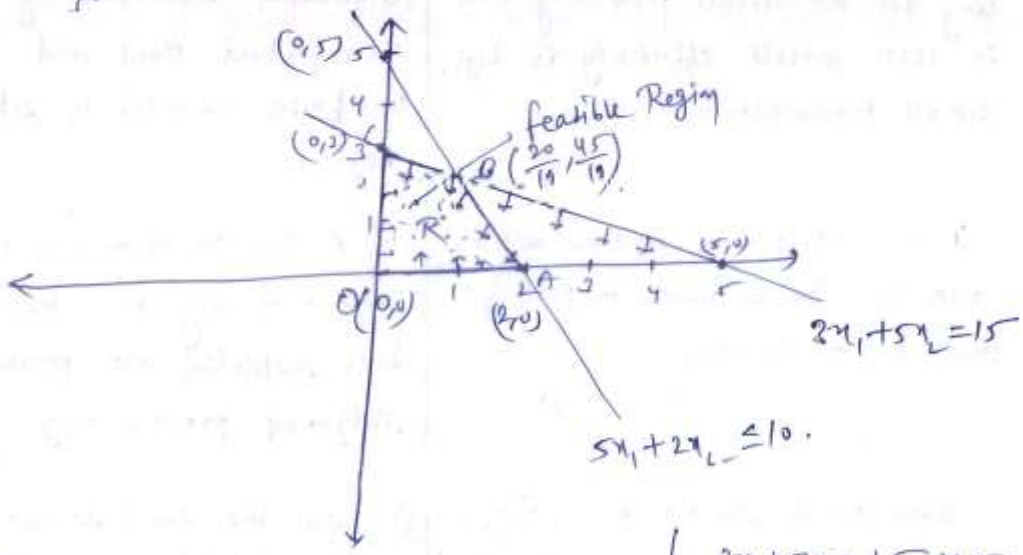
$\text{Max } Z = 5x_1 + 3x_2$

s.t.

$3x_1 + 5x_2 \le 15$ $\quad (5,0), (0,3)$

$5x_1 + 2x_2 \le 10$ $\quad (2,0), (0,5)$

$x_1, x_2 \ge 0$.



at $(0,0)$, $\text{Max } Z = 0$

at $(2,0)$ $\text{Max } Z = 10$

at $\left(\frac{20}{19}, \frac{45}{19}\right)$, $\text{Max } Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19}$

$= \frac{100}{19} + \frac{135}{19} = \frac{235}{19} = 12.36$

at $(0,3)$, $\text{Max } Z = 9$

$3x_1+5x_2=15 \times 5$

$5x_1+2x_2=10 \times 3$

$\therefore 19x_2 = 45$

$x_2 = \frac{45}{19}$

$x_1 = \frac{15 - 5 \times \frac{45}{19}}{3} = \frac{15 - \frac{225}{19}}{3}$

$= \frac{60}{19}$

So the optimal sol. is (6)

$$x_1^* = \frac{20}{19}, \quad x_2^* = \frac{45}{19},$$

$$\text{Max } Z = 12.36 \quad \text{Ans}$$

Q. (6)

Assignment problem	Transportation problem
(i) Assignment means allocating various jobs to various people in the organization. Assignment should be done in such a way that the overall processing time is less, overall efficiency is high, overall productivity is high, etc.	(i) A Transportation problem is concerned with transportation method or selecting routes in a product distribution network among the manufacture plant and distributing warehouse situated in different regions.
(ii) In an assignment problem only one allocating can be made in particular row or a column.	(ii) A transportation problem is not subject to any such restrictions. Such restriction are peculiar to assignment problems only.
(iii) When no. of jobs no. of workers is not-equal, it is a unbalanced problem.	(iii) When the total demand is not equal to total supply, it is a unbalanced problem.

Section B

Q.① (7)

	D_1	D_2	D_3	Available units	Row penalty
O_1	[5] 2	7	4	5/0	(2) × × ×
O_2	3	[2] 3	[6] 1	8/2/0	(2) (2) (2) (2)
O_3	5	[7] 4	7	7/0	(1) (1) (3) (3)
O_4	[2] 1	6	[14] 2	14/12/0	(1) (1) (4) ×
Demand	7/2/0	9/7/0	18/6/0	34	
Column penalty	(1) (2) × × ×	(1) (1) (1) (1)	(1) (1) (1) (6) ×		

Total Cost :- $(5\times2) + (2\times3) + (6\times1) + (7\times4) + (2\times1) + (12\times2)$

$= 10 + 6 + 6 + 28 + 2 + 24$

$= 76$

Optimality Test :- here Total No. of allocations is exactly

$m + n - 1 = 4 + 3 - 1 = 6$

		$V_1=0$ D_1	$V_2=3$ D_2	$V_3=1$ D_3
$U_1=2$	O_1	[5] 2	7	4
$U_2=0$	O_2	3	[2] 3	[6] 1
$U_3=1$	O_3	5	[7] 4	7
$U_4=1$	O_4	[2] 1	6	[12] 2

for unoccupied cells using formula.

$$d_{ij} = c_{ij} - (u_i + v_j)$$

$d_{12} = 7 - 5 = 2$

$d_{13} = 1$

$d_{21} = 3$

$d_{31} = 4$

$d_{33} = 5$

$d_{42} = 2$

here all $d_{ij} \geq 0$. So the optimal sol. is.

$O_1 \xrightarrow{2 \text{ units}} D_1$

$O_2 \xrightarrow{2 \text{ units}} D_2$

$O_2 \xrightarrow{6 \text{ units}} D_3$

$O_3 \xrightarrow{7 \text{ units}} D_2$

$O_4 \xrightarrow{2 \text{ units}} D_1$

$O_4 \xrightarrow{12 \text{ units}} D_3$

Total Cost: 76 Rs.

Rajasthan Institute of Engineering & Technology, Jaipur

University Roll No. _____

1st year MBA. Semester II I Mid Term Examination, March- 2018

Subject: -Operations Research **SET-B**

Time: - 2 Hrs. [Maximum Marks: -20]

[Min. Passing Marks: 08]

Instructions to the Candidates:

Attempt any 4 questions from Section A and Section B is Compulsory.

Section A

1. Sole the following problem using graphical method:

$$\text{Maximise}: Z = 5x + y$$

$$\text{subject to}: x + y \le 4;$$

$$3x + 8y \le 24$$

$$10x + 3y \le 20;$$

$$x \ge 0, y \ge 0$$

2. Construct the dual of the problem

$$\text{Minimize } z = x_2 + 3x_3$$

subject to

$$2x_1 + x_2 \le 2$$

$$x_1 + 2x_2 + 6x_3 \ge 5$$

$$-x_1 + x_2 + 2x_3 = 2$$

$$x_1, x_2, x_3 \ge 0$$

3 A department has five employees with five jobs to be performed. The time (in hours) each man will take to perform each job is given in the effectiveness matrix.

		Employees				
		I	II	III	IV	V
Jobs	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

4. find the initial basic feasible solution to the following transportation problem using least cost entry method

	D_1	D_2	D_3	D_4	Available units
O_1	13	11	15	40	2
O_2	17	14	12	13	6
O_3	18	18	15	12	7
Demand	3	3	4	5	

5. Old hens can be bought for Rs. 2 each but young ones Rs.5 each. The old hens lay 3 eggs per week and the young one 5 eggs per week, each being worth 30 paise. A hen costs Rs. 1 per week to feed. If a person has only Rs. 80 to spend for hens, and he has accommodation only for 20 hens in house .formulate this linear programming problem mathematically for maximum profit.

6.Define
(i)Expected Monetary Value Criterion(EMV)
(ii)Expected Value of perfect information(EVPI)

Section B

1.solve the following transportation problem for total minimum transportation cost

using Vogel'sapproximation method and Modi method

	D_1	D_2	D_3	D_4	Availalde units
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

2. A newspaper boy buys magazine for Rs. 13 each and sells them of Rs. 18 each .He can not return the unsold magazine .The past record of Sales are as follows:

No. of custumers :	23	24	25	26	27	28	29	30
No. of days :	5	10	15	30	20	10	5	5

(i)Prepare the opportunity loss table
(ii)Seiect the optimal act using expected orrortunity loss criterion.

Set – B

(1)

Q.① Sol. Max $Z = 5x + y$

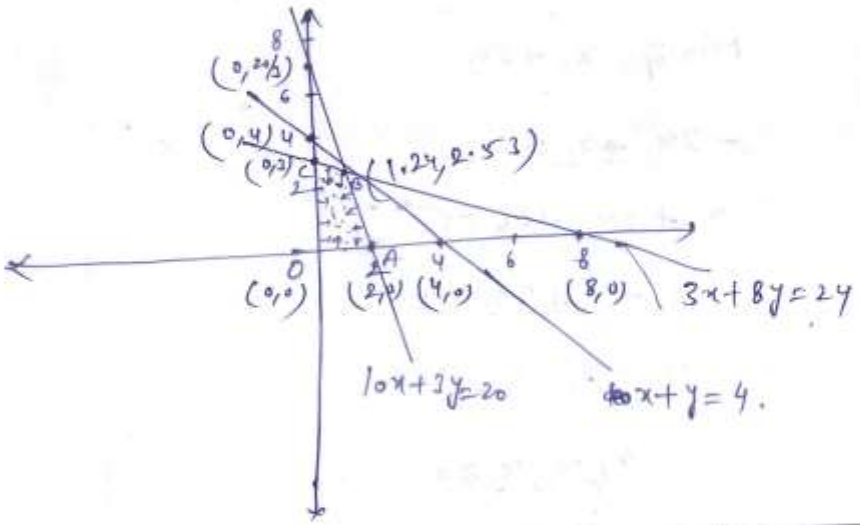
s.to. $x + y \le 4$ $\quad (4,0), (0,4)$

$3x + 8y \le 24$ $\quad (8,0), (0,3)$

$10x + 3y \le 20$ $\quad (2,0), (0, 20/3)$

$x \ge 0, y \ge 0.$

Sol



at $O(0,0)$, Max $Z = 0$

at $A(2,0)$, Max $Z = 10$

at $B(1.24, 2.53)$, Max $Z = 8.71$

at $C(0,3)$, Max $Z = 3$

optimal sol. is

$x^* = 2, y^* = 0,$ Max $Z = 10$

$3x + 8y = 24 \quad \times 10$

$10x + 3y = 20 \quad \times 8$

$80x + 80y = 240$

$80x + 24y = 160$

$56y = 80$

$y = \frac{80}{56}$

$3x = 24 - \left(8 \times \frac{180}{71}\right)$

$3x = 24 - 20.28$

$x = 1.24$

Q.(2) Sol. $\text{Min } Z = x_2 + 2x_3$

S.to.

$$2x_1 + x_2 \leq 2$$
$$x_1 + 2x_2 + 6x_3 \geq 5$$
$$-x_1 + x_2 + 2x_3 = 2$$
$$x_1, x_2, x_3 \geq 0.$$

The given problem can be written as

$$\text{Min } Z_p = x_2 + 2x_3$$
$$-2x_1 - x_2 \geq -2$$
$$x_1 + 2x_2 + 6x_3 \geq 5$$
$$-x_1 + x_2 + 2x_3 \geq 2$$
$$x_1 - x_2 - 2x_3 \geq -2$$
$$x_1, x_2, x_3 \geq 0.$$

Then its dual

$$\text{Max } Z_D = -2w_1 + 5w_2 + 2w_3 - 2w_4$$

S.to.

$$-2w_1 + w_2 - w_3 + w_4 \leq 0$$
$$-w_1 + 2w_2 + w_3 - w_4 \leq 1$$
$$0w_1 + 6w_2 + 2w_3 - 2w_4 \leq 2$$
$$w_1, w_2, w_3 \geq 0$$

Let $w_3 - w_4 = w_3'$

$$\text{Max } z_D = -2w_1 + 5w_2 + 2w_3'$$

s.to. $-2w_1 + w_2 - w_3' \le 0$

$-w_1 + 2w_2 + w_3' \le 1$

$0w_1 + 6w_2 + 2w_3' \le 3$

$w_1, w_2 \ge 0$, w_3' is unrestricted in sign.

Q. (2) Sol:

	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

Row Reduction:

	I	II	III	IV	V
A	~~5~~	[0]	~~8~~	~~10~~	~~11~~
B	[0]	6	15	10	3
C	~~8~~	~~5~~	[0]	~~X~~	~~X~~
D	~~X~~	4	2	[0]	5
E	3	5	6	~~X~~	8

(4)

	I	II	III	IV	V
A	7	[0]	8	12	11
B	[0]	4	13	10	1
C	10	5	~~[0]~~	2	~~[0]~~
D	~~0~~	2	[0]	~~0~~	3
E	3	3	4	[0]	6

Optimal assignment is

	Cost
A → II	5
B → I	3
C → V	2
D → III	9
E → IV	4
Total Cost	23 Rs. Ay

Q. (4)

	D_1	D_2	D_3	D_4	Available Units
O_1	~~13~~	[2] ~~11~~	~~15~~	~~40~~	~~2~~/0
O_2	[1] 17	[1] 14	[4] 12	13	6/~~2~~/1/0
O_3	[2] 18	18	15	[5] 12	7/~~2~~/0
Demand	3/~~2~~/0	3/1/0	4/0	5/0	15

Total Cost: $22+17+14+48+36+60$

$= 197$ Rs.

Q.(5) Let the number of hens purchased be $= x_1$ old and x_2 (young)

No of eggs received per week $= 3x_1 + 5x_2$

Money spent on feeding $= (x_1 + x_2)$ Rs.

Net profit per day $= z = \frac{3}{10}(3x_1 + 5x_2) - (x_1 + x_2)$

$$\text{Max } z = .1x_1 + .5x_2$$

S.to. Constraint

$$-2x_1 + 5x_2 \leq 80$$

$$x_1 + x_2 \leq 20$$

$$x_1, x_2 \geq 0.$$

Q.(6) (i) EMV:- In this Method first a payoff table is Constructed and then probabilities are assigned to the various events.

EMV is calculated for each act by multiplying the payoff values by corresponding probabilites and add the resulting values to obtain EMV of act and Choose highest EMV.

(ii) EVPI :- It is a maximum amount decision maker would be willing to pay to acquire perfect information as to which event would occur.

$$EVPI = EPPI - EMV$$

where $EPPI = \sum$ (Best payoff for ith event) $\times P(E_i)$

$$EMV(a_j) = \sum_{i=1}^{m} V(a_j, E_i) \cdot P(E_i)$$

Section B

Q. ①

	D_1	D_2	D_3	D_4	Availability	Row penalties
S_1	[5] M	30	50	[2] 10	9/~~4~~0	(~~9~~) (~~9~~) (10) (40)
S_2	70	30	[7] 40	[2] 60	9/2/0	(10) (20) (20) (20)
S_3	40	[8] 8	70	[10] 20	18/~~10~~/0	(12) (20) (50) X
Demand	5/0	8/0	7/0	14/~~4~~/0	34	
Column penalties	(21) (21) X X X	(22) X X X X	(10) (10) (10) (10)	(10) (10) (10) (10)		

Total Cost :- $95 + 20 + 280 + 120 + 64 + 200$

$= 779$ Rs

optimality Test :- (MODI Method)

here Total No. of allocation is exactly equal to.

$$m+n-1 = 3+4-1$$
$$= 6$$

So optimality Test is possible

		$v_1=19$ D_1	$v_2=-2$ D_2	$v_3=-10$ D_3	$v_4=10$ D_4
$u_1=0$	S_1	[5] 19	30	50	[2] 10
$u_2=50$	S_2	70	30 (+)	[7] 40	[2] 60 (−)
$u_3=10$	S_3	40	[8] 8 (−)	70	[19] 20 (+)

for unoccupied cells use formula

$$d_{ij} = c_{ij} - (u_i + v_j)$$

$$d_{12} = 30 - (-2) = 32$$

$$d_{13} = 50 - (-10) = 60$$

$$d_{21} = 70 - (69) = 1$$

$$d_{22} = 30 - (48) = -18 < 0$$

$$d_{31} = 40 - (29) = 11$$

$$d_{33} = 70$$

here $d_{22} < 0$ so this is not an optimal sol.

		$V_1=13$ D_1	$V_2=2$ D_2	$V_3=8$ D_3	$V_4=10$ D_4
$U_1=0$	S_1	[5] 19	30	50	[2] 10
$U_2=12$	S_2	70	[1] 30	[7] 40	60
$U_3=10$	S_3	40	[6] 8	70	[12] 20

$d_{12} = 32$

$d_{13} = 42$

$d_{21} = 19$

$d_{24} = 18$

$d_{31} = 11$

$d_{33} = 52$

Here all $d_{ij} \geq 0$ so optimal sol. is attended.

Total Cost : $95 + 20 + 60 + 280 + 48 + 240$

$= 743$ Rs.

Q. ②

Opportunity loss Table

Demand	Prob.	Acts 23	24	25	26	27	28	29	30
23	0.5	0	13	26	39	52	65	78	91
24	.10	5	0	13	26	39	52	65	78
25	.15	10	5	0	13	26	39	52	65
26	.30	15	10	5	0	13	26	39	52
27	.20	20	15	10	5	0	13	26	39
28	.10	25	20	15	10	5	0	13	26
29	.05	30	25	20	15	10	5	0	13
30	.05	35	30	25	20	15	10	5	0
EoL		16.25	12.15	9.85	10.25	16.05	25.45	36.65	48.75

from the values of expected opportunity loss we find that EOL for act purchase 25 magazines is least. Therefore the newspaper boys should purchase 25 magazines daily.