SOLUTION: FINITE ELEMENT METHOD, VII SEM, MECHANICAL ENGINEERING, SET-A

Q. 1 FEM: Finite element method (FEM0 is a numerical method for solving problems of engineering that are governed by elliptical partial differential equation. Upon solving these equations, FEM gives approximate solution of the problem.

Various numerical methods are used for solving these problems. Nowadays high tech packages like ANSYS, NASTRAN, ABACUS etc.. are there in market, that uses FEM to solve complex engineering problems.

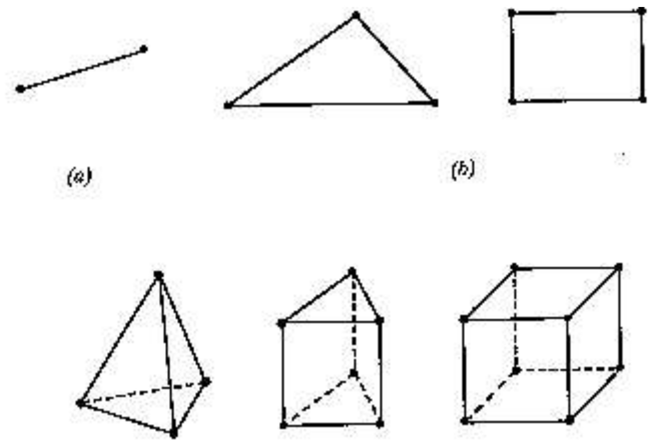
Applications:

1. Structural analysis: structural parts like, bar beam, truss etc.. are solved using FEM for design, testing of the product.
2. Non-structural analysis: Non-structural phenomenon like; heat transfer, fluid flow & electromagnetic potential etc.. are solved using FEM.
3. Other applications: Mathematical physics problems, biological problems etc…

OR

Q.1 Steps to be followed in FEM:

1. Domain-Descritization :it is the first and most important step in a FEM based analysis, because it affects the computer storage requirement, the computation time and the accuracy of the numerical results. As per the 1D, 2D or 3D domain, following types of basic finite elements can be used;



2. Selection of displacement function/interpolation functions to provide an approximation of the unknown solution with in an element.

3. Obtain the local stiffness matrices of individual elements.

4. Obtain global stiffness matrix.

5. Apply boundary conditions

6. Calculate the strains and stresses

7. Interpolation of results

Q. 2 Gauss elimination method: In linear algebra, it is a method for solving systems of linear equations by using elementary row operations till the echelon form of matrix is obtained.

Example:

$-3x + 2y - 6z = 6$
$x + 7y - 5z = 6$
$3x - 2y - 2z = 8$

Elementary row operations: R_2 ----> $3R_2 + R_1$ & R_3 -----> $R_3 + R_1$
We get,
$-3x + 2y - 6z = 6$
$23y - 21z = 24$
$-8z = 14$
On back solving,
We get,
$z = -7/4$, $y = -51/92$, $x = -105/46$

OR

Q. 2
$3x_1 + 2x_2 + x_3 = 6$
$x_1 - 10x_2 - x_3 = 2$
$-3x_1 - 2x_2 + x_3 = 0$
Elementary row operations: R_2 ----> $-3R_2 + R_1$ & R_3 -----> $R_3 + R_1$
We get,
$3x_1 + 4x_2 - 2x_3 = 6$
$32x_2 + 4x_3 = 0$
$2x_3 = 6$
On back solving,
We get,
$x_3 = 3$, $x_2 = -3/8$, $x_1 = 9/2$

Q. 3
A. Homogenous equation: $AX = 0$
Cse 1 ; rank (A) = no of the variables or $\det A \neq 0$
Trivial solution
Case 2; rank (A) < no. of variables or $\det A = 0$
Non-trivial solution
B. Non-homogenous: $AX = 0$, $B \neq 0$
Case 1; rank (A) $\neq$ rank (AB)
No solution
Case 2; rank (A) = rank (AB) = no. of variables

a. Rank (A) = rank (AB) gives Unique solution
b. Rank (A) = rank (AB) < no. of variables gives infinitely many solutions

OR

Q.3

$$\begin{matrix} 2 & 5 & 2 \\ 1 & 3 & 4 \\ 4 & 10 & 8 \end{matrix}$$

Elementary row operations: R_2 ----> $2R_2$ - R_1 & R_3 -----> R_3 - $2R_1$
We get,

$$\begin{matrix} 2 & 5 & 2 \\ 0 & -6 & -6 \\ 0 & 0 & 4 \end{matrix}$$

Which means rank of the matrix is 3

Q. 4 Stiffness: Hook's law states that the force (F) needed to extend or compress a spring by some distance 'x' scales linearly wrt that distance
i.e. f = -kx
where, k = constant factor characteristic of spring also called as stiffness]the analogue of Hook's spring law for continuous solid media
$\sigma = -E\varepsilon$
where, σ = stress, ε = displacement or strain, E = elasticity
let us take an bar element,
Element Stiffness Matrix: The stiffness matrix of a structural system can be derived by various methods like variational principle, Galerkin method etc. The derivation of an element stiffness matrix has already been discussed in earlier lecture. The stiffness matrix is an inherent property of the structure. Element stiffness is obtained with respect to its axes and then transformed this stiffness to structure axes. The properties of stiffness matrix are as follows: • Stiffness matrix is symmetric and square. • In stiffness matrix, all diagonal elements are positive. • Stiffness matrix is positive definite For example, if K is a symmetric n × n real matrix and x is non-zero column vector, then K will be positive definite while x axis positive.
Global Stiffness Matrix: A structural system is an assemblage of number of elements. These elements are interconnected together to form the whole structure. Therefore, the element stiffness of all the elements are first need to be calculated and then assembled together in systematic manner. It may be noted that the stiffness at a joint is obtained by adding the stiffness of all elements meeting at that joint. To start with, the degrees of freedom of the structure are numbered first. This numbering will start from 1 to n where n is the total degrees of freedom. These numberings are referred to as degrees of freedom corresponding to global degrees of freedom. The element stiffness matrix of each element should be placed in their proper position in the overall stiffness matrix. The following steps may be performed to calculate the global stiffness matrix of the whole structure.

a. Initialize global stiffness matrix [K] as zero. The size of global stiffness matrix will be equal to the total degrees of freedom of the structure.

b. Compute individual element properties and calculate local stiffness matrix [k] of that element.

c. Add local stiffness matrix[k]to global stiffness matrix[K] using proper locations

d. repeat the Step b. and c. till all local stiffness matrices are placed globally.

OR

Q. 4

a. Banded matrix: In matrix algebra it is a sparse matrix whose non zero entities are confined to a diagonal band, comprising the main diagonal and zero or more diagonals on either side.

If a matrix (n×n) A = a_{ij} such that a_{ij} = 0 for j < i – k_1 or j > i + k_2 ; k_1, k_2 > 0 then k_1 is lower bandwidth, k_2 is upper bandwidth.

Example: $k_1 = k_2$; diagonal matrix

$K_1 = k_2 = 1$; tridiagonal matrix

$K_1 = k_2 = 2$; penta diagonal matrix

b. Initial value problem: It is the constraint on condition given while solving ordinary differential equal such that;

y(t=0) = y_0 and y'(t=0) = y_0'

where 't' can be time or place

it gives a unique solution

Boundary value problem: If condition given is;

y(t=0) = y_0 and y(t=6) = y_1

or y'(t=0) = y_0' and y(t=6) = y_1'

t = time or space

it gives many solutions

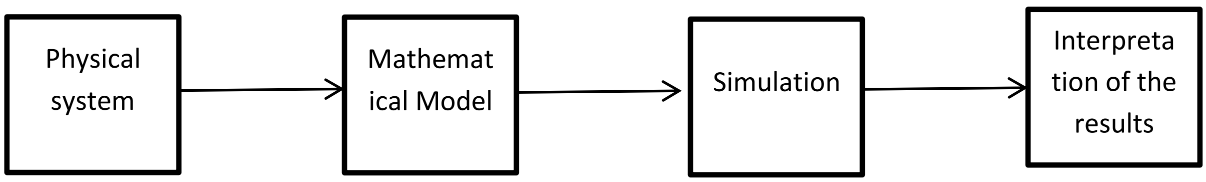
SOLUTION: FINITE ELEMENT METHOD, VII SEM, MECHANICAL ENGINEERING, SET-B

Q. 1 Numerical method is a mathematical designed to solve numerical problems with the help of various iterative method tha helps to get a converged solution which is near to to exact solution under acceptable limits of accuracy.

1. Computational Fluid Dynamics (CFD): Mainly for non-structural analysis; like heat heat flow, fluid flow; openFOAM, ANSYS FLUENT
2. Multiphysics: for the coupling of structural and non structural analysis at the same time, ANSYS, COMSOL multiphysics
3. Finite element analysis: Mainly for the structural analysis; ANSYS APDL, ABACUS
4. Language tool for coding: for the algorithm of numerical methods, MATLAB, MATHEMATICA

OR

Q.1 Steps to be followed in a numerical method



1. Physical system: First step is to understand physical problem; nature of the object(solid or fluid), type of input parameters(temperature, velocity, forces, stress, heat flux etc..).
2. Mathematical model: Find equations and theory that describe the behaviour of physical problem ex; beam theory, Navier-Stokes equations.
3. Simulation: It includes modelling of physical system, descritization, selection of proper method for analysis and then solve the given problem.
4. Interpretation of the results: After simulation desired results are displayed via graphs, contour and diagrams.

Q. 2 Order of the highest ordred non-zero minor is called as rank of the matrix.

2

Q.2 Ranking method:

The order of highest ordered non-zero minor is called as rank of the matrix.

ex. $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 3 & 2 & 1 & 3 \\ 2 & 4 & 3 & 2 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

after following row/column elementary operation

$R_2 \rightarrow R_2 - 3R_1$

$R_3 \rightarrow R_3 - 2R_1$

$R_4 \rightarrow R_4 - (R_1 + R_2 + R_3)$

We get,

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here as per definition of rank of matrix. non-zero minor is

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$ which is of order '3' i.e. matrix of 3x3 order.

Hence rank of the matrix is '3'.

OR

Q.2

$5x_1 - 4x_2 + x_3 = 10$
$x_1 - 8x_2 - 2x_3 = 4$
$-5x_1 + 4x_2 + x_3 = 2$

Elementary row operations: R_2 ----> $R_1 - 5R_2$ & R_3 -----> $R_3 + R_1$

We get,

$-5x_1 - 4x_2 + x_3 = 10$
$36x_2 + 11x_3 = -10$
$+2x_3 = 12$

On back solving,

We get,

$x_3 = 6$, $x_2 = -19/9$, $x_3 = 112/9$

Q. 3

A. Homogenous equation: AX = 0

Cse 1 ; rank (A) = no of the variables or detA $\neq$ 0

Trivial solution

Case 2; rank (A) < no. of variables or detA = 0

Non-trivial solution

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$$\begin{matrix} 3 & 4 & 3 \\ 1 & 3 & 5 \\ 6 & 8 & 8 \end{matrix}$$

Elementary row operations: R_2 ----> $R_1 - 3R_2$ & R_3 -----> $2R_1 - R_3$

We get,

$$\begin{matrix} 3 & 4 & 3 \\ 0 & -5 & -12 \\ 0 & 0 & -2 \end{matrix}$$

Which means rank of the matrix is 3

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i.e. f = -kx

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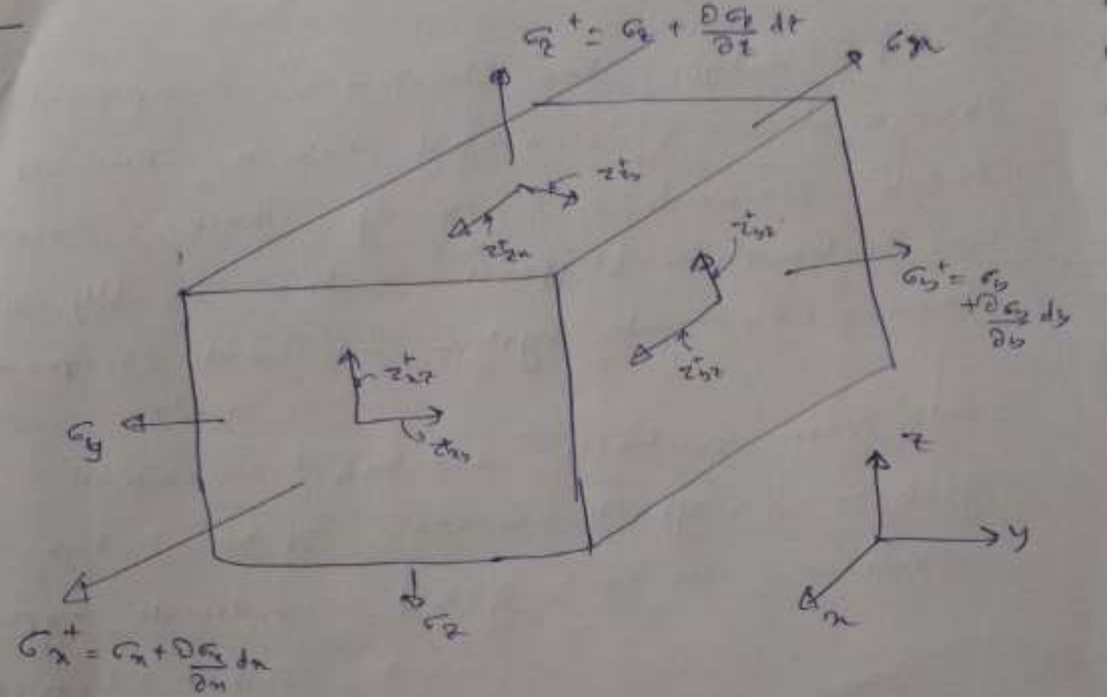
Q.4



(a)

$$\left.\begin{aligned} \tau^+_{xy} &= \tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\,dx \\ \tau^+_{xz} &= \tau_{xz} + \frac{\partial \tau_{zx}}{\partial x}\,dx \end{aligned}\right\} \text{ zy plane.}$$

$$\left.\begin{aligned} \tau^+_{zx} &= \tau_{zx} + \frac{\partial \tau_{zx}}{\partial x}\,dz \\ \tau^+_{zy} &= \tau_{zy} + \frac{\partial \tau_{zy}}{\partial z}\,dz \end{aligned}\right\} \text{ xy plane.}$$

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