Rajasthan Institute of Engineering & Technology,Jaipur.

Department of Mechanical Engineering

I Mid Term examination

Session: 2018-19

III Semester & ME

Engineering Mechanics (3ME3-04)-Solution

Time: 2hrs. M.M.:20

Instruction for students:

1. No provision for supplementary answer book.

Sec-A

Ans.01**PARALLELOGRAM LAW OF FORCES**

It states, "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."Mathematically, resultant force.

Ans.2 **LAMI'S THEOREM**

It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two." Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where, P, Q, and R are three forces and α, β, γ are the angles

Ans3 **THEOREM OF PARALLEL AXIS**

It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I G , then moment of inertia of the area about any other axis AB, parallel to the first, and

at a distance h from the centre of gravity is given by:

$$I_{AB} = I_G + ah^2$$

where I AB = Moment of inertia of the area about an axis AB,

l G = Moment of Inertia of the area about its centre of gravity

a = Area of the section, and

h = Distance between centre of gravity of the section and axis AB.

Ans.4 Angle of Friction and coefficient of friction

Angle of Friction Consider a body of weight W resting on an inclined plane that the body is in equilibrium under the action of the following forces :

1. Weight (W) of the body, acting vertically downwards,
2. Friction force (F) acting upwards along the plane, and
3. Normal reaction (R) acting at right angles to the plane.

Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which abody just begins to slide down the plane, is called the angle of friction.

This is also equal to the angle, which the normal reaction makes with the vertical.

Coefficient of friction It is the ratio of limiting friction to the normal reaction, between the two bodies, and is generally denoted by μ. Mathematically, coefficient of friction,

$$\mu = \frac{F}{R} = \tan \phi \qquad \text{or} \qquad F = \mu R$$

where φ = Angle of friction, F = Limiting friction, and R = Normal reaction between the two bodies.

Ans.5 **Velocity ratio of lifting Machine**

The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort (y) to the distance moved by the load (x) and is always expressed in pure number. Mathematically, velocity ratio,

$$\text{V.R.} = \frac{y}{x}$$

Y = Distance moved by the effort, in lifting the load, and
x = Distance moved by the load.

Sec-B

Ans.1

Solution. The system of given forces is shown in Fig. 3.12.

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = P - 3P = -2P \qquad \ldots(i)$$

and now resolving all forces vertically,

$$\Sigma V = 2P - 4P = -2P \qquad \ldots(ii)$$

We know that magnitude of the resultant forces,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-2P)^2 + (-2P)^2}$$

$$= 2\sqrt{2}P \quad \textbf{Ans.}$$

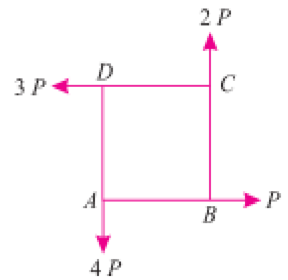


Fig. 3.12.

Direction of the resultant force

Let θ = Angle, which the resultant makes with the horizontal.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-2P}{-2P} = 1 \quad \text{or} \quad \theta = 45°$$

Since ΣH as well as ΣV are –ve, therefore resultant lies between 180° and 270°. Thus actual angle of the resultant force = 180° + 45° = 225° **Ans.**

Position of the resultant force

Let x = Perpendicular distance between A and the line of action of the resultant force.

Now taking moments of the resultant force about A and equating the same,

$$2\sqrt{2}P \times x = (2P \times a) + (3P \times a) = 5P \times a$$

$$\therefore \quad x = \frac{5a}{2\sqrt{2}} \quad \textbf{Ans.}$$

Ans.2

Solution. As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles *ABCH* and *DEFG* as shown in Fig 6.10.

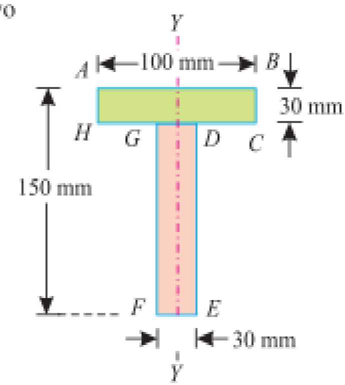


Fig. 6.10.

Let bottom of the web *FE* be the axis of reference.

(*i*) *Rectangle ABCH*

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

and
$$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(*ii*) *Rectangle DEFG*

$$a_2 = 120 \times 30 = 3600 \text{ mm}^2$$

and
$$y_2 = \frac{120}{2} = 60 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange *FE*,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} \text{ mm}$$

$$= 94.1 \text{ mm} \quad \textbf{Ans.}$$

Ans.3 **laws of static friction**

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.

2. The magnitude of the force of friction is exactly equal to the force, which tends to move the body.

3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces. Mathematically :

$$\frac{F}{R} = \text{Constant}$$

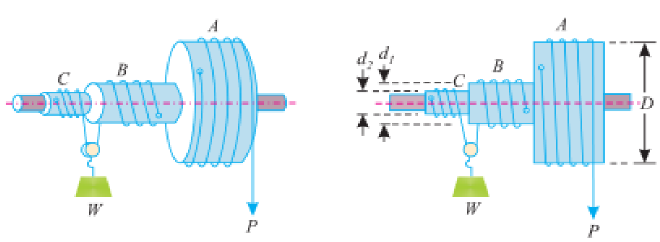
where F = Limiting friction, and

R = Normal reaction.

4. The force of friction is independent of the area of contact between the two surfaces.

5. The force of friction depends upon the roughness of the surfaces.

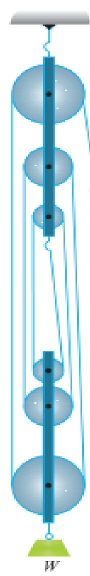
Ans.4 **Differential wheel and axle**



It is an improved form of simple wheel and axle, in which the velocity ratio is intensified with the help of load axle. A differential wheel and axle. In this case, the load axle BC is madeup of two parts of different diameters. Like simple wheel and axle, the wheel A, and the axles B and C are keyed to the same shaft, which is mounted on ball bearings in order to reduce the frictional resistance to a minimum.The effort string is wound round the wheel A. Another string wound round the axle B, which after passing round the pulley (to which the weigt W is attached) is wound round the axle C in opposite direction to that of the axle B; care being taken to wind the string on the wheel A and axle C in the same direction. As a result of this, when the string unwinds from the wheel A, the other string also unwinds from the axle C. But it winds on the axle B

Ans.5 **Second system of pulleys**

Second system of pulleys containing two blocks, one upper and the other lower, both carrying either equal number of pulleys or the upper block may have one pulley more than the lower one. the upper block is fixed and the lower one is movable. There is obly one string, which passes round all the pulleys one end of which is fixed to the upper block (when both the blocks have the same no. of pulleys) or to the lower block (when the upper block has one pulley more than the lower one). The other end of the string is free and the effort is applied to this free end In both the cases the load is attached to lower block.



Ans.6 Given: Weight (W) = 1 kN = 1000 N ; Effort (P) = 25 N ; Distance through which the weight is moved (x) = 100 mm = 0.1 m and distance through which effort is moved (y) = 8 m. We know that mechanical advantage of the machine

$$\text{M.A.} = \frac{W}{P} = \frac{1000}{25} = 40 \qquad \textbf{Ans.}$$

Velocity ratio of the machine

We know that velocity ratio of the machine

$$\text{V.R.} = \frac{y}{x} = \frac{8}{0.1} = 80 \qquad \textbf{Ans.}$$

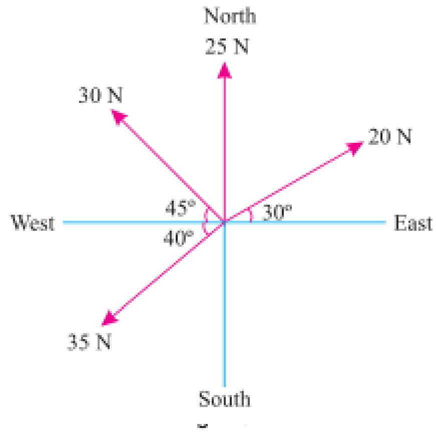
Efficiency of the machine

We also know that efficiency of the machine,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{40}{80} = 0.5 = 50\% \qquad \textbf{Ans.}$$

Sec-C

Ans.01



Magnitude of the resultant force

Resolving all the forces horizontally *i.e.*, along East-West line,

$$\Sigma H = 20 \cos 30° + 25 \cos 90° + 30 \cos 135° + 35 \cos 220° \text{ N}$$
$$= (20 \times 0.866) + (25 \times 0) + 30(-0.707) + 35(-0.766) \text{ N}$$
$$= -30.7 \text{ N} \quad \ldots(i)$$

and now resolving all the forces vertically *i.e.*, along North-South line,

$$\Sigma V = 20 \sin 30° + 25 \sin 90° + 30 \sin 135° + 35 \sin 220° \text{ N}$$
$$= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35(-0.6428) \text{ N}$$
$$= 33.7 \text{ N} \quad \ldots(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the East.

We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{33.7}{-30.7} = -1.098 \quad \text{or} \quad \theta = 47.7°$$

Since ΣH is negative and ΣV is positive, therefore resultant lies between 90° and 180°. Thus actual angle of the resultant = 180° – 47.7° = 132.3° **Ans.**

Ans.2 **Moment of Inertia of a rectangular section**

Consider a rectangular section *ABCD* as shown in Fig. 7.2 whose moment of inertia is required to be found out.

Let b = Width of the section and

d = Depth of the section.

Now consider a strip *PQ* of thickness dy parallel to *X-X* axis and at a distance y from it as shown in the figure

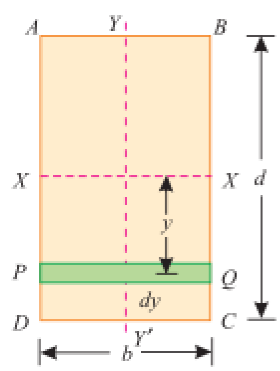
$\therefore$ Area of the strip

$= b.dy$

We know that moment of inertia of the strip about *X-X* axis,

$= \text{Area} \times y^2 = (b.\ dy)\ y^2 = b.\ y^2.\ dy$

Now *moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina *i.e.* from $-\frac{d}{2}$ to $+\frac{d}{2}$,



Rectangular section.

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b \,.\, y^2 .dy = b \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \,.\, dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right] = \frac{bd^3}{12}$$

Similarly, $\quad I_{YY} = \dfrac{db^3}{12}$

Ans.3

Solution. Given: Length of the ladder (l) = 3.25 m; Weight of the ladder (w) = 250 N; Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between the ladder and floor (μ_f) = 0.3.

Frictional force acting on the ladder.

The forces acting on the ladder are shown in Fig. 9.2.

let $\quad F_f$ = Frictional force acting on the ladder at the Point of contact between the ladder and floor, and

R_f = Normal reaction at the floor.

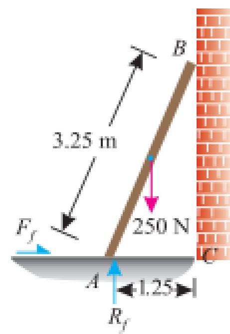


Fig. 9.2.

Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall. Resolving the forces vertically,

$$R_f = 250 \text{ N}$$

From the geometry of the figure, we find that

$$BC = \sqrt{(3.25)^2 - (1.25)^2} = 3.0 \text{ m}$$

Taking moments about B and equating the same,

$$F_f \times 3 = (R_f \times 1.25) - (250 \times 0.625) = (250 \times 1.25) - 156.3 = 156.2 \text{ N}$$

$\therefore \qquad F_f = \dfrac{156.2}{3} = 52.1\text{N} \qquad$ **Ans.**

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Sec-A

Ans.1 **POLYGON LAW OF FORCES**

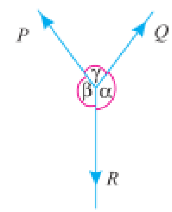
It is an extension of Triangle Law of Forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

Ans.2 **LAMI'S THEOREM**

It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two." Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where, P, Q, and R are three forces and α, β, γ are the angles



Ans.3 **THEOREM OF PERPENDICULAR AXIS**

It states, If I XX and I YY be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I ZZ about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{ZZ} = I_{XX} + I_{YY}$$

Ans.4**VARIGNON'S PRINCIPLE OF MOMENTS (OR LAW OF MOMENTS)**

It states, "If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."

Ans.5 **Law of Machine**

The term 'law of a machine' may be defined as relationship between the effort applied and the load lifted. Thus for any machine, if we record the various efforts required to raise the corresponding loads, and plot a graph between effort and load, we shall get a straight line AB the intercept OA represents the amount of friction offered by the machine.Or in other words, this is the effort required by the machine to overcome the friction, before it can lift any load.Mathematically, the law of a lifting machine is given by the relation

$$P = mW + C$$

where P = Effort applied to lift the load, m = A constant (called coefficient of friction) which is equal to the slope of the line AB,

Sec-B

Ans.1 **PARALLELOGRAM LAW OF FORCES**

It states, "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude
and direction by the diagonal of the parallelogram, which passes through their point of intersection."

Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

and
$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1).

Note. It the angle (α) which the resultant force makes with the other force F_2,

then
$$\tan\alpha = \frac{F_1 \sin\theta}{F_2 + F_1\cos\theta}$$

Cor.

1. If $\theta = 0$ *i.e.*, when the forces act along the same line, then
$$R = F_1 + F_2 \quad \text{...(Since } \cos 0° = 1)$$
2. If $\theta = 90°$ *i.e.*, when the forces act at right angle, then
$$\theta = R = \sqrt{F_1^2 + F_2^2} \quad \text{...(Since } \cos 90° = 0)$$
3. If $\theta = 180°$ *i.e.*, when the forces act along the same straight line but in opposite directions, then $R = F_1 - F_2$ ...(Since $\cos 180° = -1$)
In this case, the resultant force will act in the direction of the greater force.
4. If the two forces are equal *i.e.*, when $F_1 = F_2 = F$ then
$$R = \sqrt{F^2 + F^2 + 2F^2\cos\theta} = \sqrt{2F^2(1+\cos\theta)}$$
$$= \sqrt{2F^2 \times 2\cos^2\left(\frac{\theta}{2}\right)} \quad \text{...}\left[\because\ 1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)\right]$$
$$= \sqrt{4F^2\cos^2\left(\frac{\theta}{2}\right)} = 2F\cos\left(\frac{\theta}{2}\right)$$

:

Ans.3

Solution. As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles $ABFJ$, $EGKJ$ and $CDHK$ as shown in Fig. 6.11.

Let the face AC be the axis of reference.

(*i*) *Rectangle ABFJ*

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_1 = \frac{50}{2} = 25 \text{ mm}$$

(*ii*) *Rectangle EGKJ*

$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

and $$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

(*iii*) *Rectangle CDHK*

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_3 = \frac{50}{2} = 25 \text{ mm}$$

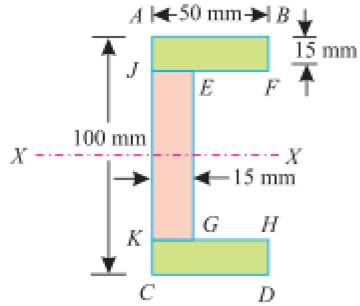


Fig. 6.11.

We know that distance between the centre of gravity of the section and left face of the section AC,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm} \quad \textbf{Ans.}$$

Ans.3 Given: Weight (W) = 1 kN = 1000 N ; Effort (P) = 25 N ; Distance through which the weight is moved (x) = 100 mm = 0.1 m and distance through which effort is moved (y) = 8 m.

We know that mechanical advantage of the machine

$$\text{M.A.} = \frac{W}{P} = \frac{1000}{25} = 40 \quad \textbf{Ans.}$$

Velocity ratio of the machine

We know that velocity ratio of the machine

$$\text{V.R.} = \frac{y}{x} = \frac{8}{0.1} = 80 \quad \textbf{Ans.}$$

Efficiency of the machine

We also know that efficiency of the machine,

$$\eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{40}{80} = 0.5 = 50\% \quad \textbf{Ans.}$$

Ans.4 **Laws of Dynamic friction**

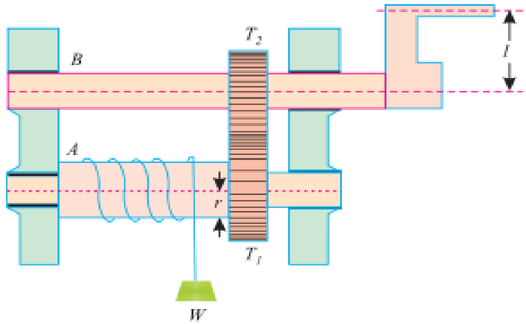
Following are the laws of kinetic or dynamic friction :

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

Ans.5 **Single purchase crab winch**

In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it.

The free end of the rope carries the load W. A toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, called pinion, is geared with the toothed wheel A The effort is applied at the end of the handle to rotate it.



Let T 1 = No. of teeth on the main gear (or spur wheel) A,
T 2 = No. of teeth on the pinion B,
l = Length of the handle,
r = Radius of the load drum.
W = Load lifted, and
P = Effort applied to lift the load.

$$\text{V.R.} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1}} = \frac{l}{r} \times \frac{T_1}{T_2}$$

Ans.6 Given: Diameter of the driven pulley (d 2) = 500 mm = 0·5 m or radius (r 2) = 0·25 m; Distance between the centres of the two pulleys (l) = 12 m and diameter of the driving pulley (d 1) = 1600 mm = 1·6 m or radius (r 1) = 0·8 m.
We know that the length of the belt if it is open,

$$L = \pi(r_1 + r_2) + 2l + \frac{(r_1 - r_2)^2}{l}$$

$$= \pi(0{\cdot}8 + 0{\cdot}25) + (2 \times 12) + \frac{(0{\cdot}8 - 0{\cdot}25)^2}{12}\ m$$

$$= 27{\cdot}32 \text{ m} \qquad \textbf{Ans.}$$

Sec-C

Ans.1

Solution. Given : Weight at C = 15 N

Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.

The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$\therefore \quad \angle ACB = 180° - (45° + 60°) = 75°$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659} = 10.98 \text{ N} \quad \textbf{Ans.}$$

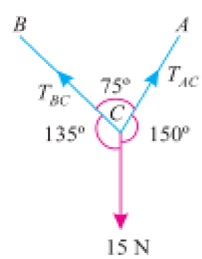


Fig. 5.4.

Ans.2 From the geometry of the truss, we find that the load of 10 kN is acting at a distance 1.25 m from the left hand support i.e., B and 3.75 m from C. Taking moments about B and equating the same,

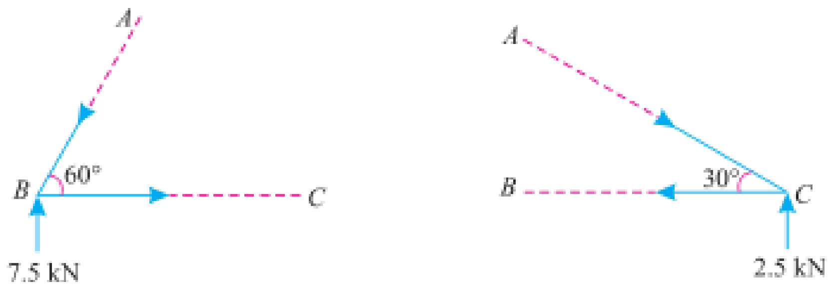
$$R_C \times 5 = 10 \times 1.25 = 12.5$$

$$R_C = \frac{12.5}{5} = 2.5 \text{ kN}$$

$$R_B = 10 - 2.5 = 7.5 \text{ kN}$$

Methods of Joints

First of all consider joint B. Let the *directions of the forces P AB and P BC (or P BA and P CB) be assumed as



(*a*) Joint *B* (*b*) Joint *C*

Fig. 13.6.

Resolving the forces vertically and equating the same,

$$P_{AB} \sin 60° = 7.5$$

or $$P_{AB} = \frac{7.5}{\sin 60°} = \frac{7.5}{0.866} = 8.66 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{BC} = P_{AB} \cos 60° = 8.66 \times 0.5 = 4.33 \text{ kN (Tension)}$$

Now consider the joint C. Let the *directions of the forces P AC and P BC (or P CA and P CB) be
be
assumed. Resolving the forces vertically and equating the same,

Now consider the joint *C*. Let the *directions of the forces P_{AC} and P_{BC} (or P_{CA} and P_{CB}) be assumed as shown in Fig. 13.6 (*b*). Resolving the forces vertically and equating the same,

$$P_{AC} \sin 30° = 2.5$$

$$\therefore \quad P_{AC} = \frac{2.5}{\sin 30°} = \frac{2.5}{0.5} = 5.0 \text{ kN (Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{BC} = P_{AC} \cos 30° = 5.0 \times 0.866 = 4.33 \text{ kN (Tension)}.$$

Ans.3 **MOMENT OF INERTIA OF A CIRCULAR SECTION**

Consider a circle $ABCD$ of radius (r) with centre O and X-X' and Y-Y' be two axes of reference through O as shown in Fig. 7.5.

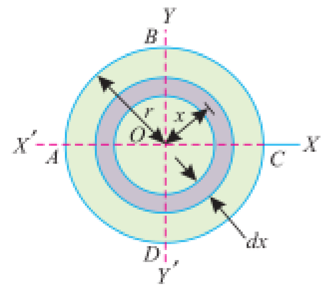


Fig. 7.5. Circular section.

Now consider an elementary ring of radius x and thickness dx. Therefore area of the ring,

$$da = 2\,\pi\,x.\,dx$$

and moment of inertia of ring, about X-X axis or Y-Y axis

$$= \text{Area} \times (\text{Distance})^2$$

$$= 2\,\pi\,x.\,dx \times x^2$$

$$= 2\,\pi\,x^3.\,dx$$

Now moment of inertia of the whole section, about the central axis, can be found out by integrating the above equation for the whole radius of the circle *i.e.*, from 0 to r.

$$\therefore \qquad I_{ZZ} = \int_0^r 2\pi x^3\,.\,dx = 2\pi\int_0^r x^3\,.\,dx$$

$$I_{ZZ} = 2\pi\left[\frac{x^4}{4}\right]_0^r = \frac{\pi}{2}(r)^4 = \frac{\pi}{32}(d)^4 \qquad \ldots\left(\text{substituting } r = \frac{d}{2}\right)$$

We know from the Theorem of Perpendicular Axis that

$$I_{XX} + I_{YY} = I_{ZZ}$$

$$* \; I_{XX} = I_{YY} = \frac{I_{ZZ}}{2} = \frac{1}{2} \times \frac{\pi}{32}(d)^4 = \frac{\pi}{64}(d)^4$$