Rajasthan Institute of Engineering & Technology,Jaipur.

I Mid Term examination

Set-A

Session: 2017-18

IV Semester

Branch –ECE

Subject with code-Advanced Engg. Mathematics-II

Time: 2hrs.

M.M.:20

Instruction for students: No provision for supplementary answer book.

Q.1 The ordinates of a normal curve are giving by the following table:

x	0	0.2	0.4	0.6	0.8
y	0.3989	0.391	0.3683	0.3332	0.2897

Evaluate (i) y(0.25) (ii) y(0.62)(iii)

OR

Q.1 Given that;

x	10°	20°	30°	40°	50°	60°	70°	80°
y	0.9848	0.9397	0.8660	0.7660	0.6428	0.5	0.3420	0.1737

Calculate y (25°), and y (73°) by using appropriate interpolation formula.

Q.2 Use Lagrange's interpolation formula to find y when x=2, given that

x	0	1	3	4
y	5	6	50	105

OR

Q.2 Define the operators and prove that relation between these operator.

Q.3 Use Picard method to solve given that y (0) = 0, compute up to fourth approximation.

OR

Q.3 Evaluate by Simpson's 1/3 rule. After finding the true value of integral ,then compute the error.

Q.4 What is relation between shift operator and inverse shift operators also prove them.!

OR

Q.4 Use Milne's method to obtain the solution of the equation $dy/dx = x - y^2$ at x=0.8 given that y(0)=0

Y(0.2)=.02, y(0.4)=.o795 y(0.6)= 0.1762, Set-B

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Q.1 Given that;

θ	$0°$	$5°$	$10°$	$15°$	$20°$	$25°$	$30°$
$\tan\theta$	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

Calculate tan 3°, tan 16°, by using appropriate interpolation formula.

OR

Q.1 A rod is rotating in a plane. The following gives the angle (in radians) through which the rod has turned for various values of time t(sec.)

T	0	0.2	0.4	0.6	0.8	1	1.2
θ	0	0.12	0.49	1.12	2.02	3.2	4.67

Calculate the angular velocity and acceleration of the rod when t = 0.2 .

Q.2 Find f(5) from the following table by using Lagrange's interpolation formula

x	1	2	3	4	7
f(x)	2	4	8	16	128

OR

Q.2 Use Milne's P-C method to solve the equation at x = 0.8

x	0	0.2	0.4	0.6
y(x)	0	0.02	0.0795	0.1762

Q.3 Use Runge-Kutta fourth order method to solve to obtain y (0.2) given that y (0) = 1 with h = 0.1.

OR

Q.3 Evaluate using (i) Trapezoidal rule(ii) Simpson's 1/3 Rule (iii) Simpson's 3/8 Rule

Q.4 Use Euler's method find the value of y at x=0.2 from the initial value problem dy/dx= 1-x+4y, y(0)= 1 taking h=0.1 .

OR

Q.4 Write down the stirling formula, simpson integration formulae, and runge kutta fourth order fourmala.

EC

Q.2. Define the operators δ and μ and prove that relation between these operators

Ans

$$\mu^2[f(x)] = \mu[\mu f(x)]$$

$$= \mu\left[\tfrac{1}{2} f(x+\tfrac{h}{2}) + f(x-h/2)\right]$$

$$= \tfrac{1}{2}\left[\mu f(x+\tfrac{h}{2}) + \mu f(x-h/2)\right]$$

$$= \tfrac{1}{2}\left[\tfrac{1}{2}\{f(x+h)+f(x)\} + \tfrac{1}{2}\{f(x)+f(x-h)\}\right]$$

$$\mu^2 f(x) = \tfrac{1}{4}[f(x+h) + 2f(x) + f(x-h)] \rightarrow (1)$$

R.H.S

$$\delta^2[f(x)] = \delta[\delta f(x)]$$

$$= \delta\left[f(x+h/2) - f(x-h/2)\right]$$

$$= \delta f(x+h/2) - \delta f(x-h/2)$$

$$= [f(x+h) - f(x)] - [f(x) - f(x-h)]$$

$$= f(x+h) - 2f(x) + f(x-h) \rightarrow (2)$$

from eq (1) & (11)

$$4\mu^2[f(x)] - \delta^2 f(x) = 4f(x)$$

$$\mu^2 f(x) = f(x) + \frac{\delta^2 f(x)}{4}$$

$$\boxed{\mu^2 \equiv 1 + \delta^2/4}$$

Q-3 Use Picard method to solve $dy/dx = x + y^2$ given that $y(0) = 0$. Compute up to fourth approximation.

Sol Picard formula is

$$y = y_0 + \int_{x_0}^{x} f(x, y)\,dx$$

$$y' = y_0 + \int_{x_0}^{x} f(x, y^0)\,dx = 0 + \int_0^x f(x, 0)\,dx$$

$$= \int_0^x x^2\,dx = x^3/3$$

$$\boxed{y' = x^3/3}$$

$$y^2 = y_0 + \int_{x_0}^{x} f(x, y')\,dx$$

$$= 0 + \int_0^x f(x, x^3/3)\,dx = 0 + \int_0^x \left(x^2 + \frac{x^6}{9}\right)dx$$

$$= \left[\frac{x^3}{3} + \frac{x^7}{67}\right]_0^x$$

$$y^2 = \frac{x^3}{3} + \frac{x^7}{63}$$

Similarly we can find y^3 & y^4

Q.3 Evaluate $\int_4^{5.2} \log_e x\,dx$ (i) Simpson 1/3rd rule. After finding the value of the Integral compare the errors in both cases.

x	$y = \log_e x$
$x_0 = 4$	1.38629
$x_0 + h = 4.2$	1.435084
$x_0 + 2h = 4.4$	1.48160
$x_0 + 3h = 4.6$	1.526056
$x_0 + 4h = 4.8$	1.5686
$x_0 + 5h = 5$	1.609437
$x_0 + 6h = 5.2$	1.6486

Q-4 what is relation between shift operator and inverse shift operators also prove that.

Sol

$$\delta = E^{1/2} - E^{-1/2}$$

$$\delta f(x) = f(x + h/2) - f(x - h/2)$$

$$\delta f(x) = E^{1/2} f(x) - E^{-1/2} f(x)$$

$$= [E^{1/2} - E^{-1/2}] f(x)$$

$$\delta = E^{1/2} - E^{-1/2} \text{ proven}$$

Similarly $\mu = \left[\dfrac{E^{1/2} + E^{-1/2}}{2}\right]$

$$\mu f(x) = \frac{f(x + h/2) + f(x - h/2)}{2}$$

$$= \tfrac{1}{2}[E^{1/2} f(x) + E^{-1/2} f(x)]$$

$$= \tfrac{1}{2}[E^{1/2} + E^{-1/2}] f(x)$$

$$\boxed{\mu = \tfrac{1}{2}[E^{1/2} + E^{-1/2}]}$$

Q-4 use Milne's method to obtain the solution of the equation $dy/dx = x - y^2$ at $x = 0.8$

given that $y(0.2) = .02$, $y(0.4) = .0795$, $y(0.6) = 0.1762$

Sol $y' = dy/dx = f(x,y) = x - y^2$

$y_0' = x_0 - y_0^2 = 0$

$y_1' = x_1 - y_1^2 = (0.2) - (.02)^2 = 0.1996$

$y_2' = x_2 - y_2^2 = (0.4) - (.0795)^2 = 0.3937$

$y_3' = x_3 - y_3^2 = (0.6) - (0.1762)^2 = 0.5689$

Using Predictor formula

$y_4 = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$

$= 0 + 4/3 \times 0.2 [2(0.1996) - (0.3937) + 2(0.5689)]$

$= \frac{0.8}{3} \times 1.1433 = 0.3049$

$y_4' = x_4 - y_4^2 = 0.8 - (0.3049)^2$

$= 0.7072$

Using Corrector formula

$y_4' = y_2 + h/3 [y_2' + 4y_3' + y_4']$

$= 0.0795 + \frac{0.2}{3}[(0.3937) + 4(0.5689) + (0.7072)]$

$= 0.3046$ Ans

simpson $1/3^{rd}$ rule.

$$\int_{4}^{5.2} \log_e x\,dx = h/3\,[y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4)]$$

$$=2/8\,[.00349529 9+4(4.57056874)$$

$$+2(3.05022046)$$

$$= 1.82784726$$

exact value of Integral.

$$\int_{4.0}^{5.2} \log_e x\,dx = [x(\log_e x-1)]_{4}^{5.2}$$

$$=1.82784741$$

Hence the error is $= .00000015$ Ans

Q-4 write down the stirling formula, simpson Integration formulae and Runge Kutta 4th order formula.

Sol. (1) Runge Kutta 4th order.

$$dy/dx = f(x,y) \qquad y = y_0 \text{ at } x = x_0$$

$$K_1 = h f(x_n, y_n)$$

$$K_2 = h f\left(x_n + h/2,\ y_n + K_1/2\right)$$

$$K_3 = h f\left(x_n + h/2,\ y_n + K_2/2\right)$$

$$K_4 = h f(x_n + h,\ y_n + K_3)$$

$$K = 1/6\,[K_1 + 2K_2 + 2K_3 + K_4]$$

$$y_{n+1} = y_n + K \quad \text{Ans}$$

(2) Simpson 1/3rd rule -

$$\int_{x_0}^{x_0+nh} f(x)\,dx = h/3\,[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})]$$

(3) Simpson 3/8th rule -

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\,[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) + 2(y_3 + y_6 + y_9 + \cdots y_{n-3})] \quad \text{Ans}$$

Q.4 Use Euler method find the value of y at $x=0.2$ from the initial value problem

$dy/dx = 1-x+4y \quad y(0)=1 \quad h=0.1$

<u>Sol</u>. Given that $f(x,y) = dy/dx = 1-x+4y$

$y_{n+1} = y_n + h f(x_n, y_n)$

put $n=0$

$y_1 = y_0 + h f(x_0, y_0)$

$= 1 + (0.1)\,[1-x_0+4y_0]$

$= 1 + (0.1)\,[1-0+4.1]$

$y_1 = 1 + (0.1)\,5 = 1.5$

Again puting $n=1$

$y_2 = y_1 + h f(x_1, y_1)$

$= (1.5) + (0.1)\,[1-x_1+4y_1]$

$y_2 = (1.5) + (0.1)\,[1-0.1+4(1.5)]$

$= 1.5 + (0.1)(1-0.1+6) = 2.19$

$y_2 = 2.19$ Ans

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Calculate tan 3°, tan 16°, by using appropriate interpolation formula.

OR

Q.1 A rod is rotating in a plane. The following gives the angle (in radians) through which the rod has turned for various values of time t(sec.)

T	0	0.2	0.4	0.6	0.8	1	1.2
θ	0	0.12	0.49	1.12	2.02	3.2	4.67

Calculate the angular velocity and acceleration of the rod when t = 0.2 .

Q.2 Find f(5) from the following table by using Lagrange's interpolation formula

x	1	2	3	4	7
f(x)	2	4	8	16	128

OR

Q.2 Find the real root of the equation by Newton-Raphson Method, correct up to four place of decimal.

Q.3 Use Runge-Kutta fourth order method to solve to obtain y (0.2) given that $\frac{dy}{dx} = x + y^2$ y (0) = 1 with h = 0.1.

OR

Q.3 Evaluate using (i) Trapezoidal rule(ii) Simpson's 1/3 Rule (iii) Simpson's 3/8 Rule

Q.4 Solve the system by Gauss seidal mrthod.

$27x+6y-z=85$ $\quad 6x+15y-2z=72$, $\quad x+y-54z=110$

OR

Q.4 Solve the system by Gauss elimination method.

$2x+4y+z=3$ $\quad 3x+2y-2z= -2$, $\quad x-y+z=6$

Absent Student on Attendance time					
S.No.	Name	Year	Room No.	Branch	Signature
1	Saurav	1	235	CSE	

EE| EEE AEM (IV) sem set B

B-1 Given that

$\theta = 0°$ $5°$ $10°$ $15°$ $20°$ $25°$ $30°$

$\tan\theta = 0$ $.0875$ $.1763$ $.2679$ $.3640$ $.4663$ 0.5774

calculat $\tan 3°$, $\tan 16°$ by using appropriate Interpolation formula

Sol Newton farward Interpolation formula

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{\lfloor 2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 y_0 + \dots$$

θ	$\tan\theta$	$\Delta\tan\theta$	$\Delta^2\tan\theta$	$\Delta^3\tan\theta$	$\Delta^4\tan\theta$	$\Delta^5\tan\theta$	$\Delta^6\tan\theta$
0	0						
		.0875					
5°	.0875		.0013				
		.0888		.0015			
10°	.1763		.0028		.0020		
		.0916		.0035		.0074	
15°	.2679		.0063		-.0054		
		.0979		-.0019			.0013
20°	.3640		.0044				
		.1023			.0063	.0117	
25°	.4663		.0088	.0044			
30°	.5774	.1111					

$x = x_0 + uh$

$3 = 0 + 5u$

$u = 3/5 = 0.6$

$$y = 0 + (0.6)\times .0875 + \frac{(0.6)(0.6-1)}{\lfloor 2}\times .0013 + \frac{(0.6)(0.6-1)(0.6-2)}{3}$$

$\times .0015 + - - - -$

solve them we get y.

Similarly we can find $\tan 16°$ by stirling formula

Q-1 A rod is rotating in a plane. the following give the angle though which the rod has turned for various value of time

T	0	0.2	0.4	0.6	0.8	1	1.2
θ	0	0.12	0.49	1.12	2.02	3.2	4.67

calculate the angular velocity and acceleration $t = 0.2$

Sol- Construct the difference table

t	θ	$\Delta\theta$	$\Delta^2\theta$	$\Delta^3\theta$	$\Delta^4\theta$	$\Delta^5\theta$
0	0					
		0.12				
0.2	0.12		0.25			
		0.37		.01		
0.4	0.49		0.26		0	
		0.63		.01		0
0.6	1.12		0.27		0	
		0.90		.01		
0.8	2.02		0.28			
		1.18				
1.0	3.2					

$$\left(\frac{d\theta}{dt}\right)_{t=0} = \frac{1}{0.2}\left[\left(\frac{.90+0.63}{2}\right) - \frac{1}{6}\left(\frac{0.01-0.01}{2}\right)\right]$$

$$= 5\left[\frac{1.53}{2}\right] = 3.825 \text{ r/sec}$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{.04}\left[0.27 - \frac{1}{12}\right]$$

$$= 6.75 \text{ rad/sec}^2$$

Q2 Find the real root of the eq by N.R.M correct up to four decimal places

$$x^3 - 3x + 8 = 0$$

Sol $f(-2) = \cancel{-8+6+8}\ 2$

$f(-3) = -14$

Let $x_1 = -2, \quad x_2 = -3$

$$x_3 = \frac{(-2)(-14) - (-3) \times 2}{(-14) - 2} = -\frac{34}{16} = -2.125$$

$f(-2.125) = .779$ (+ive)

$$x_4 = \frac{(-2.125)(-14) - (-3)(0.779)}{(-14) - 0.779}$$

$= -2.171$

$f(-2.171) = 0.2806$

$$x_5 = \frac{(2.171)(-14) - (-3)(0.2806)}{-14 - 0.2806}$$

$$x_5 = \frac{31.2358}{-14.2806} = 2.187$$

which is required value.

Q-2 Find $f(5)$ from the following table by using Lagranges Interpolation formula

x	1	2	3	4	7
$f(x)$	2	4	8	16	128

By Lagranges Interpolation formula

$$y = f(x) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2$$

$$+ \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4 + \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)}$$

$$\times 8 + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$y = \frac{\cancel{3} \times \cancel{2} \times 1 \times -\cancel{2}}{-1 \times -\cancel{2} \times -3 \times -\cancel{6}} \times \cancel{2} + \frac{8 \times 2 \times 1 \times -3}{1 \times -1 \times -\cancel{2} \times -5} \times 4$$

$$+ \frac{\cancel{4} \times 3 \times 1 \times -2}{\cancel{2} \times 1 \times -1 \times -4} \times 8 + \frac{4 \times \cancel{3} \times \cancel{2} \times -2}{\cancel{3} \times \cancel{2} \times 1 \times -3} \times 16$$

$$+ \frac{4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3} \times 128$$

$$= -2/3 + 36/8 - 24 + \frac{100}{3}$$

$$= \frac{-10 + 108 - 360 + 540}{15} = \frac{648 - 370}{15} = \frac{278}{15} \text{ Ans}$$

Q-3 use Runge-Kutta method to solve $dy/dx = x + y^2$ to obtain $y(0.2)$, $y(0) = 1$ $h = 0.1$

Sol

$K_1 = hf(x_0, y_0) = 0.1[0 + 1^2] = 0.1$

$K_2 = (0.1)\,[.05 + 1.1025] = 0.11525$

$K_3 = 0.1\,[0.05 + 1.1185] = 0.11685$

$K_4 = 0.1\,[0.1 + 1.2474] = 0.13474$

$K = 1/6\,(K_1 + 2K_2 + 2K_3 + K_4) = \frac{0.6991}{6} = 0.1165$

$y_1 = y_0 + K = 1.1165$

II step $x_1 = 0.1$ $y_1 = 1.1165$

$K_1 = 0.1\,[0.1 + 1.2466] = 0.1347$

$K_2 = 0.1\,[0.15 + 1.4014] = 0.1551$

$K_3 = 0.1\,[0.15 + 1.4259] = 0.1576$

$K_4 = 0.1\,[0.2 + 1.6233] = 0.1823$

$K = 1/6\,[0.1551 + 2(0.1551) + 2(0.1576) + (0.1823)] = 0.1571$

$y_2 = 1.1165 + 0.1571 = 1.2736$

Ans

Q-3 Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ (1) Simpson $1/3^{rd}$ rule (2) $3/8^{th}$ (3) Trapezoidal rule.

Sol

By Let $y = f(x) = \frac{1}{1+x^2}$

x	y
x_1	$\frac{1}{1+0} = 1$
x_2	$\frac{1}{1+1} = 0.5$
x_3	0.200
x_4	0.100
x_5	0.058824
x_6	0.027027

(i) BY simpson $1/3^{rd}$ rule

$$\int_0^6 f(x)\,dx = h/3\,[(y_0+y_n) + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+y_6+\cdots y_{n-2})]$$

$$= \frac{1}{3}\,[(1+.027027) + 4(.500+.100+0.038462) + 2(0.200+0.058824)]$$

$$= 1.366174$$

Similarly we can find by $3/8^{th}$ rule.

Ans 1.357082

Q-4 Solve the system by Gauss Seidal method

$27x + 6y - z = 85$ $\quad 6x + 15y - 2z = 72$

$x + y - 54z = 110$

Sol $x = \frac{1}{27}[85 - 6y + z] \rightarrow (1)$ $\quad y = \frac{1}{15}[72 - 6x - 2z]$

$z = \frac{1}{54}[110 - x - z]$

put y & $z = 0$

$x_1 = \frac{85}{27} = 3.15$

put $x = 3.15$, $z = 0$

$y_1 = 1/15 [72 - 6 \times 3.15 - 2 \times 0) = 3.54$

put $x = 3.15$

$y_1 = 3.15$

$z_1 = \frac{1}{54}[110 - 3.15 - 3.54] = 1.91$

Again taking x_1, y_1, z_1 the initial value we get we can find, x_2, y_2 & z_2

$x_2 = 2.426$, $y_2 = 3.57$ $\quad z_2 = 1.92$

$x_3 = 2.426$ $\quad y_3 = 3.572$ $\quad z_3 = 1.926$ Ans

Q-4 Solve the system by Gauss elimination method

$2x + 4y + z = 3 \quad 3x + 2y - 2z = 2 \quad x - y + z = 6$

Sol the given system of eq can be rearranged

$$x - y + z = 6$$
$$2x + 4y + z = 3$$
$$3x + 2y - 2z = -2$$

we eliminate x_1 from the eq (1) & (3)

$$6y - z = -9$$
$$y - z = -4$$
$$z = 3$$

$$x - y + z = 6$$
$$6y - z = -9$$
$$z = 3$$

$x = 2 \quad y = -1 \quad z = 3$ Ans

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x	0	0.2	0.4	0.6	0.8
y	0.3989	0.391	0.3683	0.3332	0.2897

Evaluate (i) y(0.25) (ii) y(0.62)

OR

Q.1 Given that;

x	10°	20°	30°	40°	50°	60°	70°	80°
y	0.9848	0.9397	0.8660	0.7660	0.6428	0.5	0.3420	0.1737

Calculate y (25°), y (32°), by using appropriate interpolation formula.

Q.2 Use Lagrange's interpolation formula to find y when x=2, given that

x	0	1	3	4
y	5	6	50	105

OR

Q.2 Use Regula-falsi method to solve, correct up to fourth place of decimal.

Q.3 Solve by the Modified Euler's method to solve determine y for

x = 1.1, to 1.4 by taking h = 0.1.

OR

Q.3 Evaluate using (i) Trapezoidal rule(ii) Simpson's 1/3 Rule (iii) Simpson's 3/8 Rule

Hence obtain the approximate value of π in each case.

Q.4 Fit the second degree parabola to the following data,.

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

OR

Q.4 Find the cube polynomial which takes the following value

x	0	1	2	3
y	1	2	1	10

Sub EEE/EE AEM Set A

Q-1 The ordinates of a normal curve are given by the following table

x	0	0.2	0.4	0.8	0.8
y	.3989	0.3910	0.3683	.3332	0.2897

evaluate (1) $y(0.25)$ (2) $y(0.62)$.

$$y_n = y_n + U\Delta y_0 + \frac{U(U-1)}{L2}\Delta^2 y_0 + \frac{U(U-1)(U-2)}{L3}\Delta^3 y_0 + \cdots$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	
0	.3989				
		-.008			
0.2	.3910		-0.015		
		-.023		.003	
0.4	.3683		-0.12		.00
		-.035		.004	
0.6	.3332		-.008		
		-.043			
0.8	.2897				

$$U = \frac{.25 - 0.2}{0.2} = 0.25$$

$$y(0.25) = 0.931 + 0.25(-0.023) + \frac{(0.25)(.025-1)}{L2}(-0.012)$$

$$= 0.391 - .00575 + .001125 + .0021875$$

$$= 0.30659775 \text{ Ans}$$

Backward formula - $M = \frac{0.65 - 0.6}{0.2} = 0.25$

$$y(0.65) = 0.333 + 0.25(-0.035) + 0.25\frac{(0.25+1)}{L2}$$

$$\times(-0.012) + 0.25\frac{(0.25+1)(0.25+2)}{L3} \times .003$$

$$= 0.33 - 0.00875 - .001875 + .00035156$$

$$= 0.32272656 \text{ Ans}$$

Q-1 Given that

x	10°	20°	30°	40°	50°	60°	70°	80°
Y	~~.3989~~ .9848	~~.3891~~ .9397	.8660	0.7660	.6428	0.5	.3420	0.1737

Calculate $y(25°)$, $y(32°)$

Sol

Y	X	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 x$
.9848	10°							
		-.0451						
.9397	20°		-.0286					
		-.0737		.0023				
.8660	30°		-.0263		.0008			
		-.1000		.0031		-.0003		
.7660	40°		-.0232		.0005		.0006	
		-.1232		.0036		.0003		-.0012
.6428	50°		-.0196		.0008		-.0006	
		-.1428		.044		-.0006		
0.5	60°		-.0152		.0005			
		-.158		.0049				
.3420	70°		-.0103					
		-.1683						
.1737	80°							

$$U = \frac{25-20}{10} = 0.5$$

$$Y(25°) = 0.9397 + .03685 + .00328 + .00019 - .000019$$

$$= 0.9063$$

Similarly we can find $y(32)$ by Newton forward interpolation formula.

Q-2 Use Lagranges formula find y when $x=2$

x	0	1	3	4
y	5	6	50	105

Sol

$$Y(x) = \frac{(2-1)(2-3)(2-4)}{(0-1)(0-3)(0-4)} \times 5 + \frac{(2-0)(2-3)(2-4)}{(1-0)(1-3)(1-4)} \times 6$$

$$+ \frac{(2-0)(2-1)(2-4)}{(3-0)(3-1)(3-4)} \times 50 + \frac{(2-0)(2-1)(2-3)}{(4-0)(4-1)(4-3)} \times 105$$

$$= -5/6 + 4 + 100/3 - 105/6 = 19$$

Q-2 Use Regula-falsi to solve $x^3-3x-5=0$ correct up to fourth place of decimal

Sol. Let $f(x) = x^3-3x+4=0$

$f(-2)=2$

$f(-3)=-14$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2)-f(x_1)} = \frac{(-2)(-14)-(-3)(2)}{(-14)-2}$$

$$= -34/16 = -2.125$$

$f(-2.125) = 0.779$

$$x_4 = \frac{(-2.125)(-14)-(-3)(0.779)}{(-14)-0.779}$$

$$= -2.171$$

$f(-2.171) = 0.2806$

Similarly we can find $x_5 = 2.187$ Ans

Q-3 Evaluate $\int_0^1 \frac{1\,dx}{1+x^2}$ (i) Trapezoidal rule
(ii) Simpson rule $\frac{1}{3}$rd
(iii) " " $\frac{3}{8}$th

Hence obtain the value of π in each case.

Sol $f(x) = \frac{1}{1+x^2}$ $h = \frac{1-0}{6} = 1/6$

x	0	1/6	2/3	3/6	4/6	5/6	6/6
$y = f(x)$	1.00	.97297	.9000	.8000	.69231	.59016	.5000

By simpson's 1/3 rule.

$$\int_0^1 \frac{1\,dx}{1+x^2} = h/3\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= 1/18\left[(1 + .0.5000) + 4(.97297 + .8000 + .59016) + 2(.9000 + 0.69231)\right]$$

$$= 0.785397$$

By trapezoidal rule

$$\int_0^1 \frac{1\,dx}{1+x^2} = \frac{h}{2}\left[(1 + 0.5000) + 2(0.97297 + 0.9000 + .8000 + .6723 + 0.59016)\right]$$

$$= \frac{1}{12}\left[(1.5000) + 2(3.93544)\right]$$

$$= \frac{1}{12}\left[1.5 + 7.87088\right] = 0.76091$$

by actual Integration

$$\int_0^1 \frac{1\,dx}{1+x^2}\,dx = \left[\tan^{-1}\right]_0^1$$

$$= \tan^{-1} 1 = \pi/4$$

$\pi/4 = 0.785397$ $\pi = 3.141588$

Q-4 Fit the second degree parabola to the following data

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

sol Let $Y = a + bX + cX^2$

$\Sigma Y = na + b\Sigma X + c\Sigma X^2$

$\Sigma XY = a\Sigma X + b\Sigma X^2 + c\Sigma X^3$

$\Sigma X^2Y = a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4$

x	y	X	Y	XY	X^2	X^2Y	X^3	X^4
0	1	-2	1	-2	4	1	-8	16
1	1.8	-1	1.8	-1.8	1	1.8	-1	1
2	1.3	0	1.3	0	0	0	0	0
3	2.5	1	2.5	2.5	1	2.5	1	1
4	6.3	2	6.3	12.6	4	25.2	8	16
10	12.9	0	12.9	11.3	10	30.5	0	34

$12.9 = 5a + 10c$

$11.3 = 10b$

$30.5 = 10a + 34c$

$a = 1.9 \qquad b = 1.13 \qquad c = 0.34$

$Y = 1.9 + 1.13X + 0.34X^2$

put $X = x - 2 \qquad Y = y$

$y = 1.9 - 1.07x + 0.34x^2$ Ans

Q-4 Find the cube polynomial which takes the following data

x	0	1	2	3
y	1	2	1	10

Sol

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$$y_n = f(x) = 1 + x(1) + \frac{x(x-1)}{\lfloor 2}(-2) + \frac{x(x-1)(x-2)}{\lfloor 3}12$$

$$= 1 + x - \frac{2x(x-1)}{2} + 12\frac{x(x-1)(x-2)}{\lfloor 3}$$

$$y_n = f(x) = 2x^3 - 7x^2 + 6x + 1 \quad \text{Ans}$$