Rajasthan Institute of Engineering & Technology,Jaipur.

I Mid Term examination

Session: 2017-18

Semester -II & Branch – common for all

Subject with code – Engineering Mathematics-II (MA-102)

SET –A

Time: 2hrs. M.M.:20

Instruction for students: Question paper contains two sections. Sec A- compulsory (which includes 8 short answers type questions of 0.5 marks each). Sec B- contains 06 Questions out of which any 04 questions to be attempt by the student (4 marks each).

SEC- A

Q.1 (a) Explain Echelon form of a matrix.

(b)Write Various steps to find an inverse of a matrix by E-transformation.

(c) State Cayley- Hamilton theorm

(d) Explain properties of Eigen Values and Eigen vectors.

(e)Define Periodic function with Example.

(f) Define even and odd function with suitable example.

(g) Explain fourier series formula for an even function.

(h) Define half range fourier Sine Series.

SEC –B

Q.2 Find out for what values of λ the equations $x + y + z = 1, x + 2y + 4z = \lambda$ and $x + 4y + 10z = \lambda^2$ have a solution and solve them completely in each case.

Q.3 Verify Cayley-Hamilton theorem for the following matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Q.4 Find the Eigen values and Eigen vectors of the following matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

Q.5 Reduce the matrix $\begin{bmatrix} 0 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ in its normal form and hence find its rank.

Q.6 Find the Fourier series for $f(x) = x + x^2, -\pi < x < \pi$

Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots\ldots$

Q. 7 Find the half range cosine series for the function $f(x) = (x-1)^2,\ 0 < x < 1$,

Hence show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots\ldots\ldots\ldots$

Solutions : Engg. Mathematics - II

Set - A.

(1)

Q.(1) (a) Explain Echelon form of a Matrix.

Sol. A Matrix is said to be in echelon form if it satisfies following properties

(i) Any row which has all its elements zero occurs below a row which has a non zero element.

(ii) The number of zeros before the first non zero element in a row should be less than such number of zeros in the next row.

(b) Write various steps to find an inverse of a Matrix by E- transformation.

Sol. Step I :- Write $[A : I_n]$ where I_n is the unit Matrix of order n.

Step II Apply E - row (or column) transformation to find $[I_n : B]$. Doing this we shall be able to convert A into I_n and I_n into B, which is the inverse of A.

(c) State Cayley Hamilton Theorem.

Sol. Every square Matrix satisfies its own characteristic equation. i.e.

If $|A - \lambda I| = (-1)^n [\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \cdots + a_n] = 0$

is the characteristic equation of n^{th} order square Matrix A, then

$$\left[A^n + a_1 A^{n-1} + a_2 A^{n-2} + - - - + a_n I\right] = 0.$$

(d) Explain properties of Eigen values and Eigen vectors.

Sol. (i) Matrices A and A^T have the same Eigen values.

(ii) The sum of the Eigen values of Matrix A is equal to sum of the elements of the principal diagonal of A.

(iii) If the Eigen values of A are $\lambda_1, \lambda_2, ---, \lambda_n$, then the Eigen values of A^2 will be $\lambda_1^2, \lambda_2^2, ---, \lambda_n^2$.

(iv) The eigenvectors X of a Matrix A is not unique.

(V) If $\lambda_1, \lambda_2, ---, \lambda_n$ be the distinct eigen values of an n-square Matrix A then corresponding eigen vectors $X_1, X_2, -- X_n$ form a linearly independent set

(e) Define periodic function with example

Sol. A function $f(t)$ is said to be periodic if $f(t+p) = f(t)$ for all t and for some positive integer p. p is known as period of $f(t)$.

for Example $f(t) = \sin 2t$ is a periodic function with Period $p = \pi$ for all values of t.

i.e. $f(t+\pi) = \sin[2(t+\pi)] = \sin(2t+2\pi) = \sin 2t = f(t)$

(f) Define even and odd function with suitable example.

Sol. Even function :- A function $f(x)$ is known as even function if $f(-x) = f(x)$ for all values of x.

for Example : x^2, $\cos x$, $e^x + e^{-x}$, $x^4 + \cos 2x + 2$ etc.

Odd function:- A function $f(x)$ is known as odd function if $f(-x) = -f(x)$ for all values of x.

for Example : x^3, x, $\sin x$ etc.

(g) Explain fourier series formula for an even function

Sol. If $f(x)$ is an even function between the interval $-\pi$ to π, then

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{---(1)}$$

where

$$\left.\begin{aligned} a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{0}^{\pi} f(x)\,dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_{0}^{\pi} f(x)\cos nx \end{aligned}\right\} \quad \text{---(2)}$$

(h) Define half range fourier sine series.

Sol. Half range sine series for the interval $(0, \pi)$

(4)

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{--- (1)}$$

Where $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\, dx$ --- (2)

Q. (2) Find out for what values of λ the equation

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

have a solution and Solve them Completly in each case

Sol: Augmented Matrix

$$C = [A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{array}\right]$$

The given system of equation will have a solution when $\rho(A) = \rho(C)$

It is possible only when $\lambda^2 - 3\lambda + 2 = 0$.

$$(\lambda-1)(\lambda-2) = 0$$

$$\lambda = 1, 2.$$

for $\lambda = 1$

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + y + z = 1$$

$$y + 3z = 0$$

$$\Rightarrow y = -3z$$

$$\Rightarrow x = 1 + 3z - z = 1 + 2z.$$

Let $z = K$

$x = 1+2k, y = -3k$

Where k is any arbitrary constant.

for $\lambda = 2$

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + y + z = 1$$

$$y + 3z = 1$$

$$\Rightarrow y = 1 - 3z$$

$$x = 1 + 3z - 1 - z$$

$$x = 2z$$

let $z = k$

$x = 2k, \; y = 1 - 3k$

Where k is any arbitrary constant.

Q.(B) verify Cayley-Hamiton Theorem for the matrix.

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Sol. Characteristic equation of A. is

$$|A - \lambda I| = 0.$$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0.$$

$$\Rightarrow (2-\lambda)\left[(2-\lambda)^2-1\right] + 1\left[-(2-\lambda)+1\right] + \left[1-(2-\lambda)\right] = 0.$$

$$\Rightarrow (2-\lambda)^3 - (2-\lambda) + 2 - 2(2-\lambda) = 0.$$

$$\Rightarrow (2-\lambda)^3 - 3(2-\lambda) + 2 = 0.$$

$$\Rightarrow 8-\lambda^3 - 6\lambda(2-\lambda) - 6 + 3\lambda + 2 = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0. \quad \text{———} \; (1)$$

Which is the required characteristic equation of A

Now we have to prove that A satisfies (1) i.e.

$$A^3 - 6A^2 + 9A - 4 = 0 \quad \text{———} \; (2)$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A \cdot A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

from (2)

$$A^3 - 6A^2 + 9A - 4I_3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$- 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0.$$

Hence Cayley Hamilton Theorem verified.

Q. (4) Find Eigen values and Eigen vectors of the following Matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Sol. Characteristic equation of A is $|A - \lambda I| = 0$.

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0.$$

$\Rightarrow \lambda^3 - 18\lambda^2 + 45\lambda = 0.$

$\Rightarrow \lambda = 0, 3, 15$

Eigen values of are $\lambda_1 = 0$, $\lambda_2 = 3$, $\lambda_3 = 15$.

Now the eigen vector corresponding to $\lambda = 0$ is the Solution vector of the following system of equation

$$[A - \lambda I] X_1 = 0$$

or $[A - 0I] X_1 = 0$ where $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x_1 - 6x_2 + 2x_3 = 0 \quad \text{— ①}$$

$$-6x_1 + 7x_2 - 4x_3 = 0 \quad \text{— ②}$$

$$2x_1 - 4x_2 + 3x_3 = 0 \quad \text{— ③}$$

from ② and ③

$$\frac{x_1}{21 - 16} = \frac{x_2}{-8 + 18} = \frac{x_3}{24 - 14}$$

$$\frac{x_1}{5} = \frac{x_2}{10} = \frac{x_3}{10}$$

Hence $X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ (10)

Now eigen vector corresponding to $\lambda = 3$.

$$[A - 3I] X_2 = 0 \qquad \text{where } X_2 = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_4 - 6x_5 + 2x_6 = 0 \quad \text{--- (4)}$$

$$-6x_4 + 4x_5 - 4x_6 = 0 \quad \text{--- (5)}$$

$$2x_4 - 4x_5 + 0x_6 = 0 \quad \text{--- (6)}$$

from (5) and (6)

$$\frac{x_4}{-16} = \frac{x_5}{-8} = \frac{x_6}{24-8}$$

$$\frac{x_4}{-16} = \frac{x_5}{-8} = \frac{x_6}{16}$$

Hence $X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

Q.(5) Reduce the Matrix

$$A=\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$$ in its normal

form and hence find its rank.

Sol: Given

$$A=\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$A=\begin{bmatrix}1 & 0 & 1 & 1\\ 0 & 1 & -3 & -1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$$

$R_3 \to R_3 - 3R_1, \quad R_4 \to R_4 - (R_1 + R_2)$

$$A=\begin{bmatrix}1 & 0 & 1 & 1\\ 0 & 1 & -3 & -1\\ 0 & 1 & -3 & -1\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$A=\begin{bmatrix}1 & 0 & 1 & 1\\ 0 & 1 & -3 & -1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$C_3 \to C_3 + 3C_2,\ C_4 \to C_4 + C_2$

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - C_1,\ C_4 \to C_4 - C_1$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$\rho(A) = 2.$

Q.6 Find the fourier series for $f(x) = x + x^2;\ -\pi < x < \pi.$

Hence show that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + - - -$

Sol. fourier series for $f(x)$ be

$$x + x^2 = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad \text{--- (1)}$$

Where $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (x + x^2)\, dx$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} x\, dx + \frac{1}{2\pi} \int_{-\pi}^{\pi} x^2\, dx$$

$$= \frac{2}{2\pi}\int_0^{\pi} x^2\,dx \qquad \left[\because \frac{1}{2\pi}\int_{-\pi}^{\pi} x\,dx = 0 \text{ as } x \text{ is odd}\right.$$

$$a_0 = \frac{1}{\pi}\left[\frac{\pi^3}{3}\right] = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \underset{\text{I}}{x^2}\,\underset{\text{II}}{\cos nx}\,dx \qquad \left[\because x\cos nx \text{ is an odd function}\right.$$

$$= \frac{2}{\pi}\left[\left\{x^2\,\frac{\sin nx}{n}\right\}_0^{\pi} - 2\int_0^{\pi} x\,\frac{\sin nx}{n}\,dx\right]$$

$$= -\frac{4}{n\pi}\left[\int_0^{\pi} \underset{\text{I}}{x}\,\underset{\text{II}}{\sin nx}\,dx\right]$$

$$= \frac{-4}{n\pi}\left[\left\{-x\,\frac{\cos nx}{n}\right\}_0^{\pi} + \int_0^{\pi}\frac{\cos nx}{n}\,dx\right]$$

$$= -\frac{4}{n\pi}\left[-\frac{\pi(-1)^n}{n} + \left(\frac{\sin nx}{n^2}\right)_0^{\pi}\right]$$

$$= \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x \sin nx \, dx \qquad \left[\because x^2 \sin nx \text{ is an odd function}\right]$$

$$= \frac{2}{\pi}\left[\frac{-\pi(-1)^n}{n}\right]$$

$$= -\frac{2}{n}(-1)^n$$

Hence the fourier series for $f(x)$ is given by

$$x+x^2 = \frac{\pi^2}{3} - 4\left(\cos x - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \cdots\right) +$$

$$2\left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots\right) \quad \text{—— (2)}$$

Now we have to prove that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

for this put $x = \pi$ in (2)

$$\pi + \pi^2 = \frac{\pi^2}{3} - 4\left(-1 - \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right)$$

or $\pi + \frac{2\pi^2}{3} = 4\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$ —— (3)

Also put $x = -\pi$ in (2)

$$-\pi + \pi^2 = \frac{\pi^2}{3} - 4\left(-1 - \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right)$$

or $-\pi + \frac{2\pi^2}{3} = 4\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$ —— (4).

adding ③ and ④, we get (15)

$$+\frac{4\pi^2}{3} = 8\left(1+\frac{1}{2^2}+\frac{1}{3^2}+ - -\right)$$

or $\frac{\pi^2}{6} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+ - - -$ Hence proved.

Q.⑦ Find the half range Cosine series for the function

$f(x) = (x-1)^2$; $0<x<1$.

Hence show that $\frac{\pi^2}{8} = \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+ - -$

Sol. Here half range is $0<x<1$. So the full range is $-1<x<1$. here $l=1$

Thus the half range Cosine series for $f(x)$ is given as:

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{l}\right) \quad ; \ 0<x<1$$

or $$(x-1)^2 = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\pi x) \quad \text{——} ①$$

Where $a_0 = \frac{1}{l}\int_0^l f(x)\,dx$

$$a_0 = \frac{1}{1}\int_0^1 (x-1)^2\,dx = \left[\frac{(x-1)^3}{3}\right]_0^1 = \frac{1}{3} \quad \text{——} ②$$

and $a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

$= \frac{2}{1}\int_0^1 \underset{\mathrm{I}}{(x-1)^2}\ \underset{\mathrm{II}}{\cos n\pi x}\,dx$

$= 2\left[\left\{(x-1)^2\frac{\sin n\pi x}{n\pi}\right\}_0^1 - 2\int_0^1 (x-1)\frac{\sin n\pi x}{n\pi}dx\right]$

$= \frac{-4}{n\pi}\int_0^1 \underset{\mathrm{I}}{(x-1)}\ \underset{\mathrm{II}}{\sin n\pi x}\,dx$

$= \frac{-4}{n\pi}\left[\left\{-(x-1)\frac{\cos n\pi x}{n\pi}\right\}_0^1 + \underbrace{\int_0^1 \frac{\cos n\pi x}{n\pi}dx}_{0}\right]$

$= -\frac{4}{n\pi}\left[-\frac{1}{n\pi}\right]$

$a_n = \frac{4}{n^2\pi^2}$ —— (3)

Using (2) and (3) in (1), we get

$(x-1)^2 = \frac{1}{3} + \sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi x$

or $(x-1)^2 = \frac{1}{3} + \frac{4}{\pi^2}\left[\frac{1}{1^2}\cos\pi x + \frac{1}{2^2}\cos 2\pi x + \frac{1}{3^2}\cos 3\pi x + \cdots\right]$ —— (4)

Deductions:- Put $x=0$ in (4), we get

$$1 = \frac{1}{3} + \frac{4}{\pi^2}\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

or $$\frac{2}{3} = \frac{4}{\pi^2}\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

or $$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \quad \text{——} (5)$$

Put $x=1$ in (4), we get

$$0 = \frac{1}{3} + \frac{4}{\pi^2}\left[-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right]$$

or $$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \quad \text{——} (6)$$

On adding (5) and (6), we get

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$ Hence proved.

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I Mid Term examination

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Semester -II & Branch – common for all

Subject with code – Engineering Mathematics-II (MA-102)

SET –B

Time: 2hrs M.M.:20

Instruction for students: Question paper contains two sections. Sec A- compulsory (which includes 8 short answers type questions of 0.5 marks each). Sec B- contains 06 Questions out of which any 04 questions to be attempt by the student (4 marks each).

SEC–A

Q. 1(a) Explain normal form of a matrix and describe various cases.

(b) Explain Consistency of system of linear equations.

(c) Write down various steps for finding diagonalisation of a matrix.

(d)Define Orthogonal matrix.

(e) find the formulation of fourier series for even function in the interval of (-L,L) of length 2L.

(f) Explain fourier series formula for an odd function.

(g) Explain first and second harmonic in harmonic analysis.

(h) Define half range fourier Cosine Series.

SEC –B

Q.2 Examine whether the following equations are consistent and solve them if they are consistent $x + 2y - z = 3, 3x - y + 2z = 1, 2x - 2y + 3z = 2$ and $x - y + z = -1$.

Q.3 Find the characteristic equation of A. Prove that A satisfies this euation and hence find A^{-1}.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}.$$

Q.4 Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q.5 Reduce the matrix to normal form and hence find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Q.6 Obtain the Fourier series for the function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \dfrac{\pi x}{4}, & 0 < x < \pi \end{cases}$

Q.7 Find the Fourier series to represent $f(x) = x - x^2$ in the interval $x = -1$ to $x = 1$.

[illegible]

Subject: [illegible] Engineering Mathematics [illegible]

SECTION-A

Time: [illegible] Max. [illegible]

Instructions for students: [illegible]

SEC-A

Q1. (a) Explain [illegible] and describe [illegible]

(b) Explain [illegible] system of [illegible]

(c) Write [illegible]

(d) [illegible] matrix.

(e) [illegible]

(f) Explain [illegible]

(g) Explain first and second [illegible]

(h) [illegible]

SEC-B

Q.2 [illegible]

[illegible]

Q.3 [illegible]

Set-B

Q.1 (a) Explain normal form of a Matrix and describe various Cases.

Sol. Every $m \times n$ matrix of rank $r > 0$ Can be reduced to one of the following forms, by using E-transformations.

(i) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ If $m > r$, $n < r$.

(ii) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ If $m > r, n = r$.

(iii) $[I_r \;\; 0]$ If $m = r, n < r$.

(iv) $[I_r]$ If $m = n = r$

(b) Explain Consistency of system of Linear equations.

Sol. (i) If $\rho[A:B] \neq \rho[A]$; the system is inconsistent.

(ii) If $\rho[A:B] = \rho[A] =$ number of variables; the system is Consistent and has a unique sol.

(iii) If $\rho[A:B] = \rho[A] <$ number of variables; the system is Consistent and has a infinite number of solutions.

(c) Write down various steps for finding diagonalisation of a Matrix.

Sol. Step I :- Find the Characteristic equation of given matrix A, i.e. $|A - \lambda I| = 0$.

Step II :- Find eigen values and eigen vectors corresponding to each eigen value.

Step III :- Now construct Modal Matrix B having columns as eigen vectors of Matrix A. i.e. $B = [X_1 \ X_2 \ X_3 \ \cdots X_n]$

Step IV :- Find B^{-1}.

Step V :- Now find $B^{-1}AB$, which will equal to a diagonal Matrix D, whose diagonal elements will be eigen values of Matrix A.

(d) Define orthogonal Matrix.

Sol. The Matrix A is said to be orthogonal if $AA^T = I$.

(e) Find the formulation of fourier series for even function in the interval of $(-L, L)$ of length $2L$.

Sol. $$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos \frac{n\pi x}{L}\right) \quad -(1)$$

Where $$a_n = \frac{2}{L}\int_0^L f(x) \cos \frac{n\pi x}{L}\, dx \quad -(2).$$

(f) Explain fourier series formula for any odd function

Sol. If $f(x)$ is an odd function, i.e. $a_0 = 0 = a_n$.

then fourier series is given by for the interval $(-\pi, \pi)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{--- (1)}$$

Where $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

(g) Explain first and second harmonic in harmonic Analysis.

Sol. $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

$$= \underbrace{a_0}_{\text{Constant term}} + \underbrace{(a_1 \cos x + b_1 \sin x)}_{\text{first harmonic term}} + \underbrace{(a_2 \cos 2x + b_2 \sin 2x)}_{\text{second harmonic term}} + \cdots$$

Where $a_0 = \frac{\sum f(x)}{N}$; N = Total Number of equally spaced points.

$a_1 = \frac{2}{N}\sum f(x)\cos x$

$a_2 = \frac{2}{N}\sum f(x)\cos 2x$

$b_1 = \frac{2}{N}\sum f(x)\sin x$

$b_2 = \frac{2}{N}\sum f(x)\sin 2x$

(h) Define half range fourier Cosine series.

Sol: Half range Cosine series for the interval $(0, \pi)$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{—①}$$

$$\left.\begin{aligned} \text{Where } a_0 &= \frac{1}{\pi}\int_0^{\pi} f(x)\,dx \\ a_n &= \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx \end{aligned}\right] \text{—②}$$

Q. ② Examine whether the following equations are consistent and solve them completely If they are consistent

$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$
$$x - y + z = -1$$

Sol: Augmented Matrix

$$C = [A : B]$$

$$C = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - R_1$

(5)

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right]$$

$R_3 \to 7R_3 - 6R_2$

$R_4 \to 7R_4 - 3R_2$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & -1 & -4 \end{array}\right]$$

$R_4 \to R_4 + \frac{R_3}{5}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(C) =$ Number of unknown variabu.

So the system is Consistent. and has a unique sol.

$AX = B$

$x + 2y - z = 3$

$-7y + 5z = -8$

$5z = 20$

$\Rightarrow \boxed{z = 4}$

$-7y = -8 - 20$

$-7y = -28$

$\boxed{y = 4}$

$x = 3 - 8 + 4$

$\boxed{x = -1}$

Q.③ Find characteristic equation of A. Prove that A satisfies this equation and hence find A^{-1}.

~~Sol~~

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

Sol: Characteristic equation of A.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 4-\lambda & 5 \\ 0 & -6 & -7-\lambda \end{vmatrix} = 0$$

$\Rightarrow (1-\lambda)\left[-(7+\lambda)(4-\lambda) + 30\right] - 1(-18) = 0$

$\Rightarrow (1-\lambda)\left[-(28-3\lambda-\lambda^2)+30\right] + 18 = 0$

$\Rightarrow (1-\lambda)\left[\lambda^2+3\lambda+2\right] + 18 = 0$

$\Rightarrow \lambda^3 - 3\lambda^2 - 2\lambda + \lambda^2 + 3\lambda + 2 + 18 = 0.$

$\Rightarrow \lambda^3 + 2\lambda^2 - \lambda - 20 = 0 \quad \text{——} ①$

Now we have to prove that A satisfies ①. Put $\lambda = A$ in ①

$A^3 + 2A^2 - A - 20I = 0 \quad \text{——} ②$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 6 & 6 \\ 15 & -14 & -18 \\ -18 & 18 & 19 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 6 & 6 \\ 15 & -14 & 18 \\ -18 & 18 & 19 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & -12 & -13 \\ -27 & 52 & 41 \\ 36 & -42 & -25 \end{bmatrix}$$

Substitude all the values in (2), it will be satisfies, i.e.

$$\Rightarrow \begin{bmatrix} 19 & -12 & -13 \\ -27 & 52 & 41 \\ 36 & -42 & -25 \end{bmatrix} + 2 \begin{bmatrix} 1 & 6 & 6 \\ 15 & -14 & -18 \\ -18 & 18 & 19 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix} - 20 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now to find A^{-1} Multiply equation (2) by A^{-1}, we get

$$A^2 + 2A - I - 20 \cdot A^{-1} = 0$$

or $A^{-1} = \frac{1}{20}\left[A^2 + 2A - I\right]$

$$= \frac{1}{20}\left[\begin{bmatrix} 1 & 6 & 6 \\ 15 & -14 & -18 \\ -18 & 18 & 19 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{20}\begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$$

Q.(4) Find the eigen values and Eigen vectors of the Matrix (8)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sol. The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 0-\lambda & 1 & 0 \\ 1 & 0-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-\lambda)\left[-\lambda+\lambda^2\right] - 1\left[1-\lambda\right] = 0$$

$$\Rightarrow \lambda^2 - \lambda^3 - 1 + \lambda = 0$$

$$\Rightarrow \lambda^3 - \lambda^2 - \lambda + 1 = 0$$

$$\Rightarrow \lambda^2(\lambda - 1) - 1(\lambda - 1) = 0$$

$$\Rightarrow (\lambda^2 - 1)(\lambda - 1) = 0$$

$$\Rightarrow \lambda = 1, 1, -1$$

Eigen vectors

(i) for $\lambda = 1$, Consider Matrix equation

$$[A - \lambda I] X_1 = 0 \qquad X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 + x_2 = 0$ — (1)

$x_1 - x_2 = 0$ — (2)

~~Let~~

Let $x_1 = x_2 = k = 1$ and let $z = 0$

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

(ii) for $\lambda = 1$

Again let $k = -1$ and let $z = 1$

$$X_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

(iii) for $\lambda = -1$, Consider Matrix equation $[A - \lambda I] X_3 = 0$ where $X_3 = \begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_7 + x_8 = 0$$
$$x_7 + x_8 = 0$$
$$2x_9 = 0$$

Let $x_7 = -x_8 = k = 1$ and $x_9 = 0$

So $X_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

(10)

Q.(5) Reduce the Matrix to normal form and hence find the rank of the Matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Sol. $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to -\frac{C_2}{2}$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 2 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to \frac{R_2}{2}$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & -4 & 3/2 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & -4 & 3/2 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to -\frac{R_3}{3}$

$$\begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & -4 & 3/2 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_3$, $R_2 \to R_2 + 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & -7/6 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 - \frac{5}{6} C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -7/6 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + \frac{7}{6} C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + \frac{2}{3} C_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right] = \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

$\underline{\underline{\rho(A) = 3}}$

Q.6 Obtain the fourier series for the function

$$f(x) = \begin{cases} 0 \ ; & -\pi < x < 0 \\ \frac{\pi x}{4} \ ; & 0 < x < \pi \end{cases}$$

Sol. $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ — (1)

where $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$

(1a)

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} \frac{\pi x}{4}\,dx\right]$$

$$= \frac{1}{2\pi}\cdot\frac{\pi}{4}\left(\frac{x^2}{2}\right)_0^{\pi}$$

$$a_0 = \frac{1}{8}\left(\frac{\pi^2}{2}\right) = \frac{\pi^2}{16}. \quad \text{—— (2)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\cdot\cos nx\,dx + \int_{0}^{\pi}\frac{\pi x}{4}\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\cdot\frac{\pi}{4}\int_{0}^{\pi} \underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{\cos nx}\,dx$$

$$= \frac{1}{4}\left[\left\{\frac{x\sin nx}{n}\right\}_0^{\pi} - \int_0^{\pi}\frac{\sin nx}{n}\,dx\right]$$

$$= \frac{1}{4}\left[\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{1}{4}\left(\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right)$$

$$a_n = \frac{1}{4n^2}\left[(-1)^n - 1\right]$$

$$a_n = \cancel{\frac{1}{4n^2}}\begin{cases} 0 \ ; & \text{when } n \text{ is even integer.} \\ -\frac{1}{2n^2} \ ; & \text{when } n \text{ is odd integer} \end{cases} \quad \text{— (3)}$$

~~Putting the values from ② and ③ in ①, we get~~ (14)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{\pi x}{4}\sin nx\,dx$$

$$= \frac{1}{4}\int_0^{\pi} \underset{I}{x}\,\underset{II}{\sin nx}\,dx$$

$$= \frac{1}{4}\left[\left\{-\frac{x\cos nx}{n}\right\}_0^{\pi} + \underbrace{\int_0^{\pi}\frac{\cos nx}{n^2}\,dx}_{0}\right]$$

$$= \frac{1}{4}\left[\frac{-\pi(-1)^n}{n}\right]$$

$$b_n = -\frac{\pi}{4n}(-1)^n \quad \text{—— (4)}$$

Now from (1)

$$f(x) = \frac{\pi^2}{16} - \frac{1}{2}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \text{---}\right]$$

$$- \frac{\pi}{4}\left[-\frac{1}{1}\sin x + \frac{1}{2}\sin 2x - \frac{1}{3}\sin 3x + \text{---}\right] \text{—— (4)}$$

Which is the required fourier series.

Q.7 Find the fourier series to represent $f(x) = x - x^2$ in the interval $x = -1$ to $x = 1$. (15)

Sol. here $l = 1$.

fourier series is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\pi x + b_n \sin n\pi x \quad \text{--- (1)}$$

Where $a_0 = \frac{1}{2 \cdot 1} \int_{-1}^{1} f(x)\,dx$

$$= \frac{1}{2} \int_{-1}^{1} (x - x^2)\,dx$$

$$= -\frac{2}{2} \int_0^1 x^2\,dx \qquad \left[\because \int_{-1}^{1} x\,dx = 0 \right]$$

$$a_0 = -\left(\frac{x^3}{3}\right)_0^1 = -\frac{1}{3} \quad \text{--- (2)}$$

$$a_n = \frac{1}{1} \int_{-1}^{1} (x - x^2) \cos n\pi x\,dx.$$

$$= -2 \int_0^1 \underset{\mathrm{I}}{x^2} \underset{\mathrm{II}}{\cos n\pi x}\,dx$$

$$= -2 \left[\left\{ x^2 \frac{\sin n\pi x}{n\pi} \right\}_0^1 - 2\int_0^1 x \frac{\sin n\pi x}{n\pi}\,dx \right]$$

$$= \frac{4}{n\pi} \int_0^1 \underset{\mathrm{I}}{x} \underset{\mathrm{II}}{\sin n\pi x}\,dx$$

(16)

$$= \frac{4}{n\pi}\left[\left\{-\frac{x\cos n\pi x}{n\pi}\right\}_0^1 + \underbrace{\int_0^1 \frac{\cos n\pi x}{n\pi}\,dx}_{0}\right]$$

$$= \frac{4}{n\pi}\left[-\frac{(-1)^n}{n\pi}\right]$$

$$a_n = -\frac{4}{n^2\pi^2}(-1)^n \quad \text{—— (3)}$$

$$b_n = \frac{1}{1}\int_{-1}^{1}(x-x^2)\sin n\pi x\,dx$$

$$= 2\int_0^1 \underset{\text{I}}{x}\,\underset{\text{II}}{\sin n\pi x}\,dx \qquad \left[\because \int_{-1}^{1} x^2 \sin n\pi x\,dx = 0\right]$$

$$= 2\left[\left\{-\frac{x\cos n\pi x}{n\pi}\right\}_0^1 + \underbrace{\int_0^1 \frac{\cos n\pi x}{n\pi}\,dx}_{0}\right]$$

$$= 2\left[-\frac{(-1)^n}{n\pi}\right]$$

$$b_n = -\frac{2}{n\pi}(-1)^n \quad \text{—— (4)}$$

from (1)

$$f(x) = -\frac{1}{3} - \frac{4}{\pi^2}\left[\frac{-1}{1^2}\cos\pi x + \frac{1}{2^2}\cos 2\pi x + \cdots\right] - \frac{2}{\pi}\left[\frac{-1}{1}\sin\pi x + \frac{1}{2}\sin 2\pi x + \cdots\right]$$

Which is the required fourier series.